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A NEGATIVE EXPERIMENT RELATING TO MAGNETISM AND THE EARTH'S ROTATION

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The discovery by Babcock of the magnetism of certain rapidly rotating stars led me to study the hypothesis, first clearly discussed by Schuster and by Wilson, that the magnetism of rotating astronomical bodies might be due to some new and general property of matter. The well-known theoretical difficulties attending such a view were matched by the difficulty of finding a quantitative explanation of even the earth's magnetic field in terms of the known laws of physics. A detailed study of the possibility of making a direct test of the Schuster-Wilson hypothesis, by measuring the very small magnetic field of the order of 10^{-9} G which would be produced by a rotating body of reasonable size in the laboratory, led me to conclude that the experiment would perhaps be possible but would certainly be exceedingly difficult.

However, a much easier but still worth-while subsidiary experiment presented itself. This was to test whether a massive body, in fact a 10×10 cm gold cylinder, at rest in the laboratory and so rotating with the earth, would appear to an observer, also rotating with the earth, to produce a weak magnetic field with a magnitude of the order of 10^{-8} G. That such a field might exist is a plausible deduction from a particular form of the Schuster-Wilson hypothesis considered in some detail by Runcorn and by Chapman.

This paper describes the design, construction and use of a magnetometer with which this 'static-body experiment' was carried out. Since few detailed studies of the design of sensitive magnetometers to measure steady fields appear to have been made since the days of the classical experiments of Rowland and of Eichenwald, I found it necessary to investigate the theory and use of such an instrument in considerable detail. The bulk of this paper, that is, §§ 2 to 5, is concerned with this instrumental study. The actual static-body experiment is described in § 6, and it is there shown that no such field as is predicted by the modified Schuster-Wilson hypothesis is found. This result is in satisfactory agreement with the independent refutation of the hypothesis by the measurements by Runcorn and colleagues of the magnetic field of the earth underground.

When the magnetometer was completed it was found to be very suitable for the measurement of the remanent magnetism of weakly magnetized specimens, in particular certain sedimentary rocks.

1·1. Introduction

Measurements of very weak static magnetic fields have been of historical importance in several branches of physics. Of special significance was the classical experiment of Rowland (1885), in which the magnetic field produced by a moving electrostatic charge was first demonstrated.

Speculations as to the origin of the earth's magnetic field have led several experimenters to attempt to detect a magnetic field associated with a body rotated in the laboratory. Lebedev (1912) rotated small toroidal rings of various materials at speeds up to 500 rev/s. Using an astatic magnetometer he could detect no magnetic field greater than about 10^{-3} G. In 1911 Schuster (unpublished) planned and partly carried out a similar experiment, but apparently the experiment was never completed.

Swann & Longacre (1928) spun a copper sphere of 20 cm diameter at 200 c/s, and showed that no magnetic field was produced as great as 10^{-4} G. The method of detection consisted of measuring with an amplifier and an alternating-current galvanometer the electromotive force generated in a pair of coils spinning on a single shaft and disposed near the sphere symmetrically about the diametrical plane. Swann & Longacre calculated the expected effect on various theoretical hypotheses; amongst these was the assumption that the same relation between the dipole moment and angular momentum held for the copper sphere as for the earth and sun. This hypothesis, which was derived from the work of Schuster (1912) and Wilson (1923), led to a predicted field of 3×10^{-9} G. Thus Swann & Longacre's achieved sensitivity would have to be increased in the ratio of about 10^5 to 1 in order to be significant for a test of the Schuster-Wilson hypothesis.

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The discovery, on the one hand, of the magnetism of certain stars by Babcock in 1947, and on the other the difficulties met by all existing theories of the magnetism of stellar bodies, led me to interest myself in the possibility of a decisive laboratory test of the hypothesis that a rotating body produces a magnetic field of the magnitude required to explain stellar and terrestrial magnetism. A review of the earlier literature and some discussion of the existing evidence for and against such a theory have been given by Blackett (1947, 1949 *a*).

From the calculations given in the first of these papers, it is clear that the first step in planning a decisive laboratory experiment with a rotating body must be the development of a method of detecting a static magnetic field of the order of 10^{-9} to 10^{-10} G, so allowing a field of 10^{-8} to 10^{-9} G to be measured with significant accuracy.

In this paper I shall describe in detail the theoretical and experimental design of a very sensitive astatic magnetometer, and I shall show that the desired sensitivity can be attained.

As the rotating body experiment was clearly likely to be one of great difficulty, another and much simpler experiment was planned first. This was to test whether a small magnetic field exists near a massive body at rest relative to the measuring instrument, due to the fact that both test body and measuring instrument are rotating with the earth. Such an effect would be a probable consequence of a particular form of the Schuster-Wilson hypothesis. This is that a massive body in rotation be considered the seat of a virtual current. Recent theoretical discussions of this hypothesis, have been given by Blackett (1949 *b*), Papapatrou (1950) and Luschak (1951). Assuming this, Runcorn (1948) and Chapman (1948) calculated the variation of the earth's field with depth below the surface. By measurement in mines the hypothesis was disproved by Runcorn, Benson, Moore & Griffiths (1950, 1951).

Calculation also shows that, if this theory were true, a massive body, for instance, a gold sphere 10 cm in diameter, would be expected to produce a field at its surface of the order of 10^{-8} G. The execution of this static-body experiment will be discussed in § 6, where it will be shown that no field of this order of magnitude exists. Thus these two independent results agree.

Apart from the possibility, which was still open at the time of planning the static-body experiment, that a positive result might have been obtained, the experiment provides excellent experience in the problems of making significant measurements of very small fields, and as such is a valuable preliminary to planning the rotating body experiment.

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When I had completed the magnetometer, I realized that it was in many ways admirably suited to the measurement of the magnetic properties of very weakly magnetized geological specimens, for instance, certain sedimentary rocks. Measurements of this kind have been shown to be of extreme interest, particularly through the series of researches recently carried out by Johnson (1949), Torreson (1949) and colleagues at the Carnegie Institute of Terrestrial Magnetism in Washington and by Nagata and colleagues (1943), Kawai (1951) and others in Tokyo. These measurements, which were made by spinning the specimen and measuring the alternating current induced in a coil by the permanent magnetism, have given important information on the pre-history of the earth's magnetic field.

A discussion of the sensitivity of the Carnegie type of instrument is given in appendix 3. A similar type of instrument has been used by Bruckshaw (1948) for measuring the magnetism of igneous rocks. Recently measurements have been made of sedimentary rocks by Kawai (1951), using a special type of resonant astatic magnetometer capable of great sensitivity.

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1·2. *Methods of measuring weak static magnetic fields*

There appear to be at the present time only three known methods which demand seriou consideration, the astatic magnetometer, the rotating coil and the fluxgate detector; o these the first will be shown to be much the most promising.

The fluxgate detector is admirably suited to the measurement of fields of 10^{-5} to 10^{-6} G but the ultimate limit, as set by the noise-generating properties of existing magneti materials, seems not much below 2×10^{-7} G. With present knowledge there seems n possibility of using this method to detect fields as low as 10^{-9} G. A short description of th method and references are given in § 3·6.

It is more difficult to set a limit to the sensitivity of the spinning-coil method withou some further investigations, but there are clearly severe practical difficulties. If the induce currents are led out by slip rings or commutators, contact e.m.f.'s are a serious difficulty This could, in principle, be avoided by mounting the first stages of an amplifier on th rotating system and leading out the amplified current through slip rings. However, th complication and bulk of such an arrangement make it unattractive.

An alternative method tried out by Goldsack (1949, unpublished work at Manchester) is to spin a closed copper ring about a diameter and to place near the spinning ring a pai of fixed coils. The alternating current induced in the spinning ring by the external stati field induces an alternating current of double frequency in the fixed coil, which can then b amplified. This method avoids the use of any slip rings. The ultimate limit in this method, a in the analogous direct detection of an alternating magnetic field by the current generate in a fixed coil, is set by the thermal noise of the coil and by the time of observation (se appendix 3). Using a special form of integrating device Goldsack was able to detect field of 10^{-6} G. This limit could undoubtedly have been reduced by further work, but it seeme unlikely that 10^{-9} G could easily be reached, so the work was stopped. However, the discussion of appendix 3 shows that further consideration of the method might be profitable.

The astatic magnetometer, consisting essentially of two equal and oppositely directed magnets fixed to a vertical rod suspended by a thin fibre, has been extensively used for nearly a hundred years for the detection of weak magnetic fields; as, for instance, in the classical experiment by Rowland (1885) already mentioned. Eichenwald repeated Rowland's experiment in 1903 with great precision, and it is possible to deduce from the description of the astatic magnetometer used, that he was able to detect fields of rather less than 10^{-7} G.

A perfectly astatic system, that is, one in which the resultant magnetic moment of the two magnets is exactly zero, will be undisturbed by any changing uniform magnetic field; in a non-uniform field its deflexion is proportional to the difference of horizontal field at the top and bottom magnets, and so measures the vertical gradient $\mathrm{d}H/\mathrm{d}z$ of the horizontal field.

Parastatic systems, consisting of two astatic systems one above the other, with the middle two magnets pointing in the same direction, have also been used. They are unaffected by any horizontal field with a uniform vertical gradient, and their deflexion is a measure of the second vertical gradient $\mathrm{d}^2H/\mathrm{d}z^2$ of the horizontal field.

Astatic magnet systems have also been used extensively for the detection of small electric currents as in galvanometers of the Paschen type. A discussion of some aspects of the design

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of such astatic galvanometers has been given by Daynes (1925) and Hill (1926). It can be calculated that the magnetic field which acts on the magnet system of such a galvanometer (for example, that made by the Cambridge Instrument Co.), when the minimum detectable current is flowing in the coils, amounts to about 2×10^{-8} G. However, the very small size of the suspended system leads to a large Brownian movement and so to errors of a reading at least as great as 10^{-8} G.

In the early instruments of this type only carbon steel with its relatively poor magnetic properties was available out of which to make the magnets. In order to obtain the greatest possible magnetic moment for a given moment of inertia, each magnet of an astatic pair was usually made up of a number, e.g. 3 to 6, of smaller magnets spaced suitably in a vertical direction (see § 2·5).

The degree of astaticism of the system, that is, the degree of equality and parallelism of the dipoles of the two magnets, was usually quite low, and the zero of the suspended system was mainly or wholly governed by the horizontal component of the earth's field, or the field of small control magnets acting on the resultant moment of the two magnets. In some instruments a silk fibre was used so that the control was wholly magnetic. In others for instance in that used by Barnett (1935, p. 138) a quartz fibre was used, in which case the control was partly elastic and partly magnetic.

Recently a convenient and rugged type of astatic magnetometer has been described by Johnson & Steiner (1937), which has Alnico magnets and is designed for the measurement of the magnetic properties of geological and other weakly magnetic specimens. The magnet system, which was suspended by a quartz fibre and had a period of about 7 s, was made astatic to within about 1 % by demagnetizing the stronger of the two magnets by a suitable alternating field. The sensitivity of this magnetometer, as normally used, corresponded to about 2×10^{-7} G per mm deflexion at 1 m, though the sensitivity was stated to have been capable of being increased to about 2×10^{-8} G/mm. No figure was given for the accuracy of a single determination.

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The problem arises as to what is the optimum design of a magnetometer to meet a given specification of required performance. Since this design must depend on the magnetic properties of existing magnetic materials, it became necessary to investigate the properties of typical examples of the new magnetic alloys as regards suitability for magnet construction, and to assess their relative advantages over the older material.

The problem of designing a magnetometer to give the highest possible performance can be split into two nearly separate tasks:

(*a*) The design of a suspended system to give the highest performance in the absence of external disturbances.

(*b*) The reduction to the minimum of the effects of external disturbances due to external magnetic fields and to mechanical and thermal effects. If possible such disturbances must be kept to so small a value as to leave the residual error of a reading governed only by the Brownian movement of the suspended system.

A general discussion of the effect of Brownian movement on the accuracy of measurements has been given by Barnes & Silverman (1934).

The magnetometer which will be described in this paper follows mainly the conventional pattern for astatic magnetometers. The only novelty in its design, apart from very careful

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attention to detail, appears to be in the method of astaticizing the suspended system by means of small trimming magnets. It is the very high degree of astaticism attained by this means which makes the system relatively immune to external magnetic disturbances, and so allows a high sensitivity resulting from the use of a long period of oscillation to be usefully employed.

2. Theoretical design of the magnetometer

2·1. *Fundamental relations*

Some, but not all, of the results of this section are similar to those which have been obtained by Daynes (1925) and by Hill (1926) in analyses of the sensitivity of the Paschen galvanometer. However, since the problem treated in this section is not quite the same as that treated by these two workers, no detailed reference to their results will be made.

If I_0, I_1 are the moments of inertia of a single magnet, and the mirror and the rod holding the mirror and the two magnets, then for an astatic system the total moment of inertia I is given by

$$I = 2I_0 + I_1. \tag{1}$$

It is convenient to write

$$I = \alpha I_0, \tag{2}$$

where α must be larger than 2 for an astatic system and larger than 4 for a parastatic system.

If σ is the torsional constant of the fibre and T the undamped period of the system, we have

$$T^2 = 4\pi^2 I/\sigma. \tag{3}$$

Since the system will be used in nearly the critically damped condition, T can be interpreted in what follows as approximately equal to the effective time of a single observation. For, at critical damping, the ratio of the deflexion after time T to the initial deflexion is $\exp(-2\pi) = 0{\cdot}002$. Thus, if a reading of the magnetometer deflexion is taken at a time interval T after application of the magnetic field to be measured, the error will be only 0·2%, and so, in general, is quite negligible. When a lower degree of accuracy is allowable a shorter time of observation is tolerable, e.g. a 1% error results from a time of observation of $0{\cdot}66T$.

In what follows the time T will be interpreted as either the undamped period of the suspension or the duration of an observation, according to the context.

If a magnetic field H acts at right angles to the lower magnet of dipole moment P, a deflexion θ_H is produced, given by

$$\theta_H = \frac{HP}{\sigma}, \tag{4}$$

which, using (3), gives for the sensitivity θ_H/H

$$\frac{\theta_H}{H} = \frac{T^2}{4\pi^2}\frac{P}{I}, \tag{5}$$

showing that the sensitivity is proportional to T^2 for given values of P and I, and is proportional to P/I for given T.

If $\epsilon = kt$, where k is Boltzmann's constant and t the temperature ($\epsilon = 4\times10^{-14}$ erg at 15° C), then the r.m.s. deflexion θ_0 of the suspended system is given by

$$\tfrac{1}{2}\sigma\theta_0^2 = \frac{\epsilon}{2},$$

whence

$$\theta_0 = \epsilon^{\frac{1}{2}}/\sigma^{\frac{1}{2}}. \tag{6}$$

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Then, with (3),
$$\theta_0 = \frac{\epsilon^{\frac{1}{2}} T}{2\pi I^{\frac{1}{2}}}, \tag{6a}$$

showing that the thermal 'noise' increases as T. We see, therefore, from (5) and (6a) that the 'signal-noise ratio'
$$\frac{\theta_H}{\theta_0} = \frac{H}{2\pi\epsilon^{\frac{1}{2}}} T \frac{P}{I^{\frac{1}{2}}} \tag{7}$$

is proportional to T for a given magnet, and proportional to $P/I^{\frac{1}{2}}$ for a given period.

Practical considerations connected with the effect of external disturbances, which will be discussed in §2·10, set an effective limit to the increase of the duration of observation as a way of increasing the signal-noise ratio. I have not found it possible to use successfully values of T longer than about 45 s; 30 s is generally a more convenient value.

As an alternative to making use of the signal-noise ratio, it is convenient to introduce the quantity H_0, defined as that magnetic field which will produce a deflexion equal to the r.m.s. thermal deflexion θ_0. Then we have
$$PH_0 = \sigma\theta_0, \tag{8}$$
or, using (3) and (6a),
$$H_0 = \frac{2\pi\epsilon^{\frac{1}{2}}}{T}\frac{I^{\frac{1}{2}}}{P}. \tag{9}$$

From its definition the quantity H_0 is also the standard error of a single reading of the magnetic field due to the thermal motion of the suspended system. For convenience it will be often spoken of as the minimum detectable field. From (6a) and (9) we find
$$\theta_0 = \epsilon/H_0 P. \tag{10}$$

Equation (10) gives the angular deflexion corresponding to H_0. Clearly the optical system must be such as to make θ_0 measurable.

2·2. *Variation of performance with size of magnet*

Since from (9) the minimum detectable field for given T is proportional to $I^{\frac{1}{2}}/P$, we shall first investigate how this latter quantity varies with the linear dimension l of the magnet, assuming the shape constant. As a simplification we shall assume the quantity α in (2) to be constant, so that we have $H_0 \propto I_0^{\frac{1}{2}}/P$.

Since the dipole moment of a magnet of given shape and material is proportional to its volume, we have $P \propto l^3$. Since $I_0 \propto l^5$ we have $H_0 \propto I_0^{\frac{1}{2}}/P \propto l^{-\frac{1}{2}} \propto P^{-\frac{1}{6}}$. Thus, though H_0 decreases as P is increased, the rate of variation is so slow as to make a very large increase in P necessary to give appreciable reduction in H_0. For instance, to reduce H_0 by a factor of $\frac{1}{2}$ requires a fourfold increase in l, and so a 64-fold increase of P.

There are two limitations to the useful increase of P. First, the external magnetic field of the magnetometer itself becomes large, and this introduces unwanted effects in many experiments. Secondly, as H_0 is decreased by increasing l, it can be seen from (6a) and the argument above, that $\theta_0 \propto l^{-\frac{5}{2}} \propto H_0^5$ and so decreases rapidly. For magnets larger than a certain size θ_0 becomes so small as to be optically unmeasurable without such a large increase in mirror size as to involve an unacceptable increase of its moment of inertia; for such an increase will eventually cancel the expected gain of sensitivity (§2·7).

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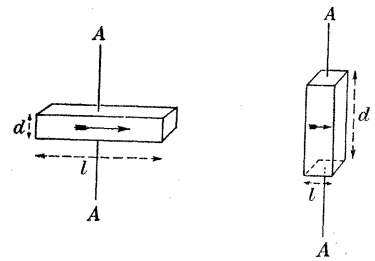
The above considerations make it clear that no unlimited reduction in H_0 can be usefully achieved either by increase in the time of observation or by increase in size of magnet. Thus it is essential to design the magnet with the lowest possible value of $I_0^{\frac{1}{2}}/P$ in order to obtain the highest accuracy of a reading.

Since from (5) the sensitivity θ/H is proportional to P/I for given T, and since $P/I \propto l^{-\frac{1}{2}}$, the *sensitivity*, but not of course the *accuracy*, can be increased indefinitely by *reducing* the size of the magnet.

2·3. *Moment of inertia of rectangular magnet of varying fineness ratio*

Since magnets made out of most modern magnetic alloys are easiest made of rectangular cross-section, we will consider a series of magnets of square section and varying length-breadth ratio (figure 1), magnetized (*a*) parallel to the long side, and (*b*) parallel to a short side.



(*a*) $\beta = l/d > 1$ (*b*) $\beta = l/d < 1$

FIGURE 1. Rectangular magnets magnetized (*a*) longitudinally, (*b*) transversely.

(*a*) The moment of inertia of a magnet of mass M and length l and width and breadth d, about an axis normal to the centre of a long face is

$$I_0 = \frac{M}{12}(l^2+d^2),$$

or, introducing the fineness ratio β defined as

$$\beta = l/d,$$

we have $$I_0 = \frac{M}{12} l^2(1+\beta^{-2}) \quad (\beta > 1). \tag{11}$$

If ρ is the density of the material, the volume $V = M/\rho$ of the magnet equals ld^2 or l^3/β^2, whence $$l^2 = (M\beta^2/\rho)^{\frac{2}{3}}. \tag{12}$$

With (11) we get $$I_0 = M^{\frac{5}{3}}\rho^{-\frac{2}{3}}f(\beta), \tag{13}$$

where $$f(\beta) = (\tfrac{1}{12})\,\beta^{\frac{4}{3}}(1+\beta^{-2}) \quad (\beta > 1). \tag{14}$$

(*b*) For a magnet transversely magnetized and suspended about an axis through the centre of its square face, we find in a similar way

$$I_0 = M^{\frac{5}{3}}\rho^{-\frac{2}{3}}f(\beta),$$

where $$f(\beta) = (\tfrac{1}{6})\,\beta^{\frac{2}{3}} \quad (\beta < 1). \tag{15}$$

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Equations (13) and (14) give then the moment of inertia of a series of magnets of varying fineness ratio β greater than unity; equation (15) gives the corresponding value of $f(\beta)$ for β less than unity. For $\beta > 1$ the magnet is longitudinally magnetized and transversely suspended; for $\beta < 1$ it is transversely magnetized and longitudinally suspended. The values of $[f(\beta)]^{\frac{1}{2}}$ are shown graphically in figure 2. From the above definitions $f(\beta)$ has a discontinuous first derivative for $\beta = 1$.

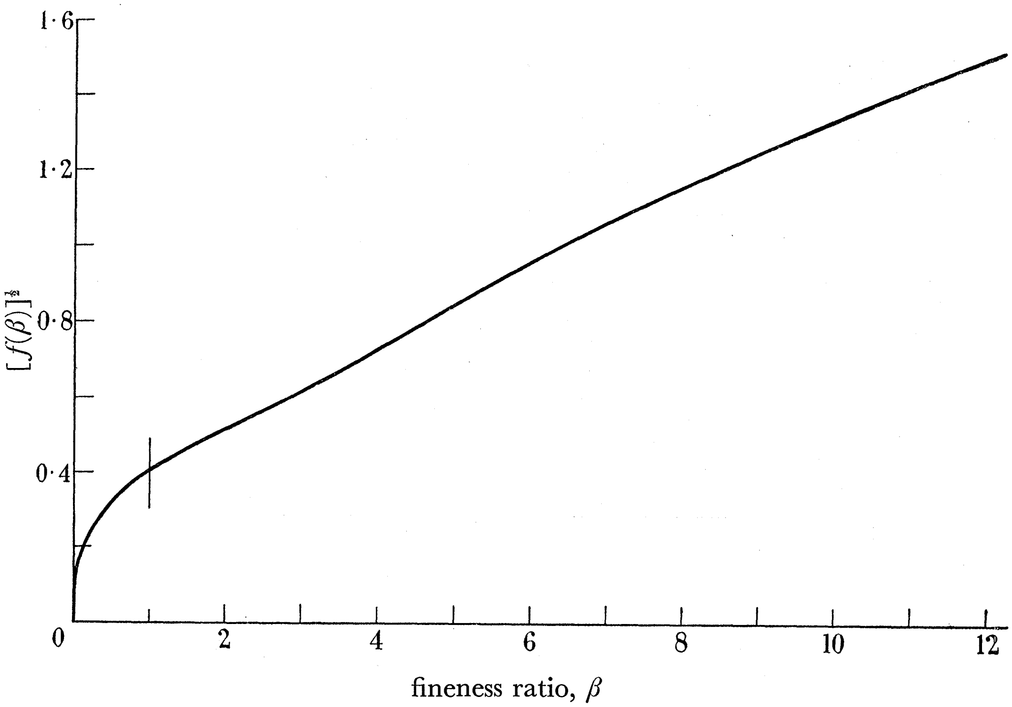

FIGURE 2. Variation of $[f(\beta)]^{\frac{1}{2}}$ with fineness ratio β. $\beta > 1$, equation (14); $\beta < 1$, equation (15).

2·4. *Design of the magnet*

When a piece of magnetic material is magnetized to saturation by an external field and is then removed from the field, the magnetization falls to some value less than the remanent intensity because of the demagnetizing effect of its own magnetization. Since this demagnetizing effect depends on the shape but not on the size of the magnet, the dipole moment P of a rectangular magnet of mass M, fineness ratio β and given material, can be written

$$P = MJ_s(\beta), \tag{16}$$

where $J_s(\beta)$ is the dipole moment for unit mass for some material S and given fineness ratio β.

The function $J_s(\beta)$ (figure 3) for any given material can be determined by direct measurement of the dipole moments of rectangular specimens. Alternatively it can be calculated approximately from the normal B-H curves, using theoretical values for the demagnetization coefficient. In table 1 are given details of some typical magnetic materials and in appendix 1 the method of calculating $J_s(\beta)$.

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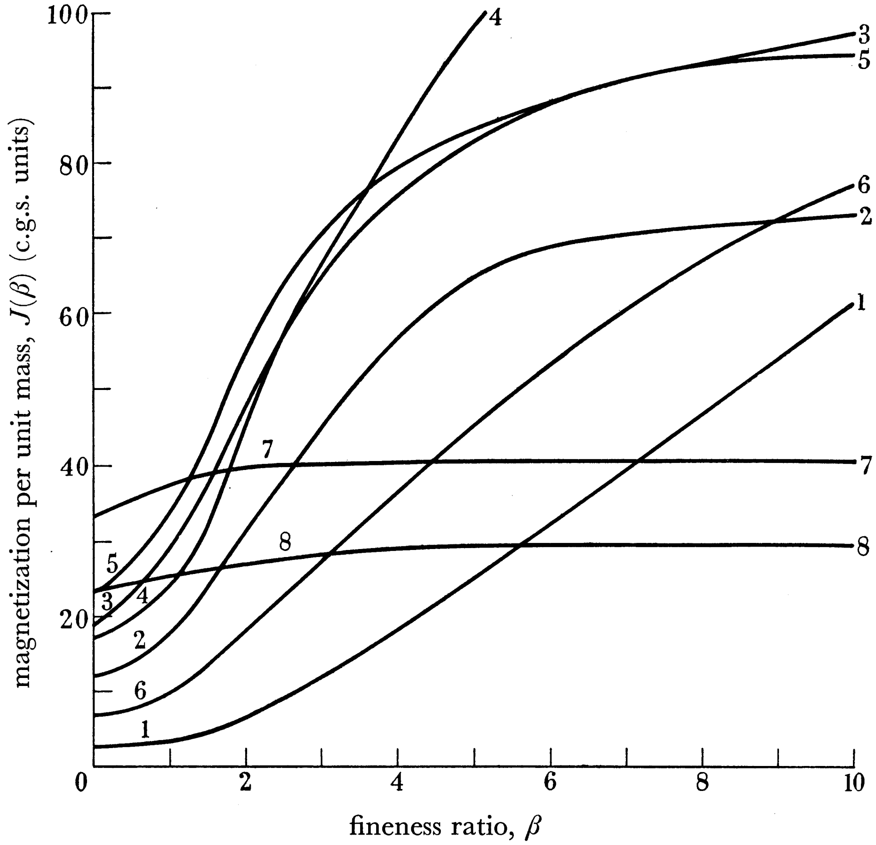

FIGURE 3. Intensity of magnetization $J(\beta)$ for the eight materials listed in table 1, plotted against the fineness ratio β.

TABLE 1. PROPERTIES OF SELECTED MATERIALS FOR PERMANENT MAGNETS

Density ρ, remanence B_r, coercive force $(B=0)$ H_c; maximum value of BH. The quantity in the last column is seen from equation (19*a*) to be proportional to the performance of a magnetometer consisting of long transversely magnetized magnets

material	composition (%)	ρ	B_r	H_c	(BH) $(\times 10^{-6})$		no. of curve figures 3, 4 and 5	$\rho^{\frac{1}{3}}[J(0)]^{\frac{5}{6}}$
15% Co steel	15 Co, 85 Fe	7·8	8200	180	0·62	machinable in annealed condition	1	4·1
Alnico	10 Al, 17 Ni, 12 Co, 6 Cu	7·5	7250	560	1·70	brittle; cast and ground to shape	2	18·4
Hycomax*	9 Al, 21 Ni, 20 Co, 15 Cu, 1·5 Ti	7·3	8500	790	2·8		3	22·1
Alcomax IV*	8 Al, 12 Ni, 24 Co, 6 Cu, 2 Cb	7·5	11200	750	4·3		4	20·5
Ticonal K*	—	7·3	9050	1080	3·96	very brittle; cast and ground to shape	5	26·2
Vicalloy	52 C, 10 V	8·1	9000	300	1·02	malleable in annealed condition	6	9·6
Vectolite*	44 Fe_3O_4, 30 Fe_2O_3, 26 Co_2O_3	3·12	1600	900	0·5	soft, graphite-like	7	27·2
Caslox	17 Co, 26 Fe, 27 O_2	3·25	1100	700	0·2		8	20·6

* Those materials marked with an asterisk are anisotropic, the rest are isotropic.

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Now from (13) and (16) we find

$$\frac{I_0^{\frac{1}{2}}}{P} = P^{-\frac{1}{6}}(J(\beta))^{-\frac{5}{6}}(f(\beta))^{\frac{1}{2}}\rho^{-\frac{1}{3}}, \tag{17}$$

whence, with (2) and (9), we have

$$H_0 = \frac{2\pi\epsilon^{\frac{1}{2}}\alpha^{\frac{1}{2}}}{TP^{\frac{1}{6}}}G_s(\beta), \tag{18}$$

where

$$G_s(\beta) = \frac{[f(\beta)]^{\frac{1}{2}}}{\rho^{\frac{1}{3}}[J_s(\beta)]^{\frac{5}{6}}}. \tag{19}$$

Equation (18) gives the minimum detectable magnetic field H_0 in terms of the thermal energy ϵ, the duration of the observation T, the strength of the magnet P, the ratio α of the moment of inertia of the whole magnetometer to that of a single magnet, and a function $G_s(\beta)$ which depends only on the magnetic properties of the material and on the fineness ratio β. To obtain the lowest value of H_0 we must choose that material and fineness ratio which gives the lowest value of $G_s(\beta)$.

In figures 4 and 5 are given the curves of $G_s(\beta)$ on two different scales for various materials, approximately calculated according to the method described in appendix 1.

Now it will be seen from figure 3 that as β approaches zero, $J_s(\beta)$ in all cases approaches a finite value $J_s(0)$. Since from (15) we have

$$f(\beta) = \tfrac{1}{6}\beta^{\frac{2}{3}}$$

for $\beta \ll 1$, we find from (19) that

$$G_s(\beta) = \frac{1}{6^{\frac{1}{2}}}\frac{\beta^{\frac{1}{3}}}{\rho^{\frac{1}{3}}[J_s(0)]^{\frac{5}{6}}}. \tag{19a}$$

From figures 4 and 5 we see that the curves for Ticonal K and Alcomax IV show shallow minima with $G(\beta)$ about 0·009 for β about 2 and 4 respectively and then drop to zero for $\beta \ll 1$. These two materials give the lowest possible value of $G(\beta)$ for longitudinally magnetized magnets. On the other hand, if it is possible to use very long and thin *transversely magnetized* rods one can obtain lower values. For instance, a Ticonal rod 20 times as long as wide ($\beta = 0{\cdot}05$) would give $G(\beta) = 0{\cdot}0057$. Unfortunately, the mechanical properties of these brittle alloys make it very difficult to manufacture such thin rods. However, they could perhaps be made in a number of pieces and then mounted in line.

It will be noticed that Vectolite, a sintered oxide powder, gives almost the same performance as Ticonal K for $\beta < 0{\cdot}25$.

Though in principle one can obtain as low a value of $G(\beta)$ as one pleases by making β small enough, the fact that, according to (19*a*), $G(\beta)$ varies only slowly with β, makes it necessary to go to very long and thin transverse magnets to gain very much. Thus to reduce β from 0·05 to 0·01 would only give a reduction of H_0 in the ratio of 1 to $1/5^{\frac{1}{3}} = 0{\cdot}59$. Such long and thin magnets could only easily be made of a ductile material such as Vicalloy which can be drawn into wire. However, existing materials of this type have appreciably worse magnetic properties than the brittle alloys or the sintered oxides. Magnets which are very long in the vertical direction are also inefficient in cases where the field to be measured decreases rapidly with height. Further, there seems some doubt if it is possible to magnetize such thin wires of isotropic material transversely.

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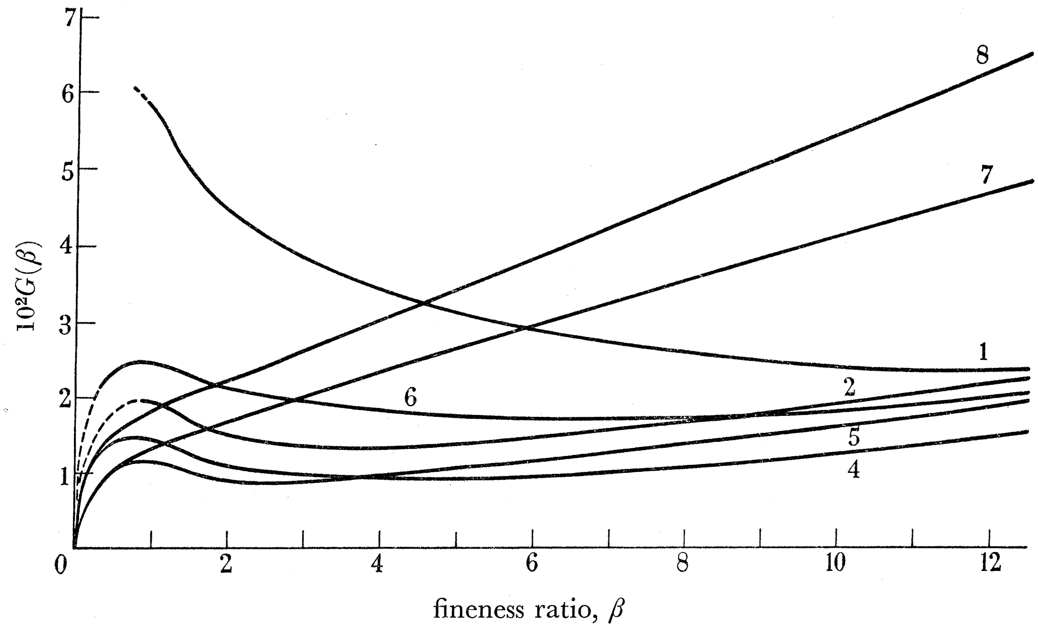

FIGURE 4. $G(\beta)$ given by equation (19) plotted against β for seven of the listed materials.

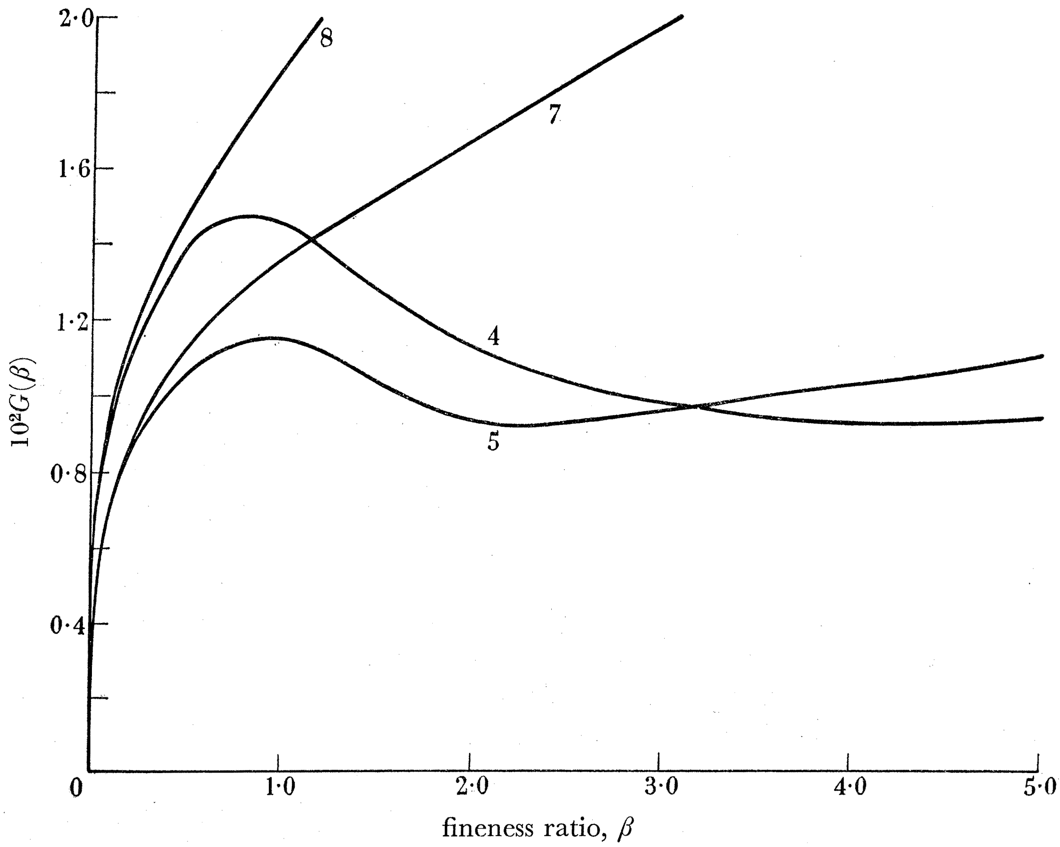

FIGURE 5. $G(\beta)$ given by equation (19) plotted on a large scale for four of the listed materials. Curve 4, Alcomax IV; curve 5, Ticonal K; curve 7, Vectolite; curve 8, Caslox.

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From (19*a*) it is seen that the relative performance of a magnetic material when used in the form of a long transversely magnetized rod, as measured by the reciprocal of the minimum detectable field, is proportional to the quantity $\rho^{\frac{1}{3}}[J_s(0)]^{\frac{5}{6}}$. This is tabulated in the last column of table 1.

We conclude, therefore, that it is not possible with existing materials to obtain conveniently values of $G(\beta)$ less than about 0·006. Since 15 % cobalt steel, which was available in 1921, gives $G(\beta) = 0{\cdot}024$ for $\beta \sim 12$, we see that the best modern magnetic materials give only the disappointingly small reduction of $G(\beta)$ from 0·024 to 0·006, that is, in the ratio 4 : 1.

2·5. *The use of multiple magnets*

The traditional method of increasing the performance of a magnetometer is to use a number of small magnets in place of one larger one. From equation (9) we see that the minimum detectable magnetic field H_0 for a single magnet at constant T is proportional to $I_0^{\frac{1}{2}}/P$. Suppose now we split the single magnet into n smaller magnets of the same shape and of magnetic moment P/n, leaving the total magnetic moment the same. Then each small magnet has linear dimensions $n^{-\frac{1}{3}}$, and so a moment of inertia $n^{-\frac{5}{3}}$ times that of the large magnet. Thus the total moment of inertia is $n \times n^{-\frac{5}{3}} I_0 = n^{-\frac{2}{3}} I_0$, and so from (9) H_0 is reduced by the factor $n^{-\frac{1}{3}}$.

For instance, if $n = 8$, H_0 is reduced by a factor 2. To reduce H_0 by a factor of 4 would need 64 magnets. Apart from the awkward constructional problem arising from such multiple magnets, the same serious difficulty arises as is mentioned in the last paragraph, that is, the length of the array of small magnets—they cannot be placed too close together owing to their mutual demagnetizing effect—becomes prohibitive. If they are placed quite close together, introducing a large mutual demagnetizing effect, the array of small magnets becomes in effect a single transversely magnetized magnet with very small β, the theory of which we have already discussed.

2·6. *The vertical distance between the magnets of an astatic system*

The distance L has not entered our calculations explicitly, as so far we have only considered a single magnet. For an astatic pair the expressions derived above remain valid if the difference ΔH of the field at the two magnets is substituted for H and the appropriate value of α is used in (2). If, as is usual, the test body of which the field is to be measured is placed close below the magnetometer, and if $H(z)$ is the horizontal field at a height z above the specimen, then

$$\Delta H = H(z_0 - \tfrac{1}{2}L) - H(z_0 + \tfrac{1}{2}L),$$

when z_0 is the distance of the specimen to the centre of the magnetometer.

In general L should be chosen large enough to make the second term small compared with the first, for instance, not more than one-quarter of it; that is, L should be comparable to the vertical extension of the field to be measured. Values of L from 3 to 20 cm are usual in magnetometers. In cases where the field to be measured has a nearly constant vertical gradient $\mathrm{d}H/\mathrm{d}z$ it is desirable to increase L as much as possible. In practice the useful increase in L is limited both by the greater fragility and moment of inertia of a very long magnetometer, and by the increased effect of external magnetic disturbances.

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2·7. *Minimum detectable magnetic field*

We shall use (18) and (10) to get some numerical values of the minimum detectable field H_0 and the angular deflexion θ_0 corresponding to this field.

Putting in the value of $\epsilon = 4 \times 10^{-14}$ erg, equation (18) gives

$$H_0 = 1{\cdot}26 \times 10^{-6} \alpha^{\frac{1}{2}} G(\beta) / TP^{\frac{1}{6}}.$$

For α the value of 3 will be assumed; this implies that the moment of inertia of mirror and rod equals that of one of the magnets. For $G(\beta)$ the value of 0·009 will be assumed in accordance with the discussion of § 2·4. Then we get

$$H_0 = 1{\cdot}96 \times 10^{-8} / TP^{\frac{1}{6}}. \tag{20}$$

In table 2 are given some approximate numerical values of H_0 for various assumed values of T and P. Table 3 gives the corresponding values of θ_0 obtained from (10).

TABLE 2. MINIMUM DETECTABLE MAGNETIC FIELD IN GAUSS H_0 CALCULATED FROM EQUATIONS (18), (19) AND (20) FOR VARIOUS VALUES OF THE PERIOD T AND THE DIPOLE MOMENT P, ASSUMING $G(\beta) = 0{\cdot}009$

T (sec) \ P (G cm³)	1000	100	10	1	10^{-1}
10	$6{\cdot}3 \times 10^{-10}$	$9{\cdot}3 \times 10^{-10}$	$1{\cdot}4 \times 10^{-9}$	$2{\cdot}0 \times 10^{-9}$	$2{\cdot}9 \times 10^{-9}$
30	$2{\cdot}1 \times 10^{-10}$	$3{\cdot}1 \times 10^{-10}$	$4{\cdot}5 \times 10^{-10}$	$6{\cdot}7 \times 10^{-10}$	$9{\cdot}8 \times 10^{-10}$
100	$6{\cdot}3 \times 10^{-11}$	$9{\cdot}3 \times 10^{-11}$	$1{\cdot}4 \times 10^{-10}$	$2{\cdot}0 \times 10^{-10}$	$2{\cdot}9 \times 10^{-10}$
300	$2{\cdot}1 \times 10^{-11}$	$3{\cdot}1 \times 10^{-11}$	$4{\cdot}5 \times 10^{-11}$	$6{\cdot}7 \times 10^{-11}$	$9{\cdot}8 \times 10^{-11}$

TABLE 3. ANGULAR DEFLEXION θ_0 IN RADIANS CORRESPONDING TO THE VALUES OF H_0 GIVEN IN TABLE 3, CALCULATED FROM EQUATION (10)

At 5 m scale distance the corresponding scale deflexion in microns is obtained by multiplying the figures for θ_0 by 10^7.

T (sec) \ P (G cm³)	1000	100	10	1	10^{-1}
10	$6{\cdot}3 \times 10^{-8}$	$4{\cdot}3 \times 10^{-7}$	$3{\cdot}1 \times 10^{-6}$	$2{\cdot}0 \times 10^{-5}$	$1{\cdot}4 \times 10^{-4}$
30	$1{\cdot}9 \times 10^{-7}$	$1{\cdot}3 \times 10^{-6}$	$9{\cdot}3 \times 10^{-6}$	$6{\cdot}0 \times 10^{-5}$	$5{\cdot}2 \times 10^{-4}$
100	$6{\cdot}3 \times 10^{-7}$	$4{\cdot}3 \times 10^{-6}$	$3{\cdot}1 \times 10^{-5}$	$2{\cdot}0 \times 10^{-4}$	$1{\cdot}4 \times 10^{-3}$
300	$1{\cdot}9 \times 10^{-6}$	$1{\cdot}3 \times 10^{-5}$	$9{\cdot}3 \times 10^{-5}$	$6{\cdot}0 \times 10^{-4}$	$5{\cdot}2 \times 10^{-3}$

It will be shown in § 5·2 that it is possible by a single visual observation to make a reading of the spot with a standard error of about 2×10^{-6} radian with a mirror 10 mm in width which has its central portion blacked out to narrow the fringes. This corresponds to detecting a linear shift of the spot of about 5 % of the width of the central fringe. Using this result and assuming, according to diffraction theory, that the minimum detectable angle is inversely proportional to the width of the mirror, the minimum mirror width is $2 \times 10^{-6}/\theta_0$ radian. The values are given in table 4 and are to be interpreted as minimum width of mirror required to allow the deflexion θ_0 corresponding to H_0 to be measured.

From the design standpoint one can make the following deductions from the figures of tables 2, 3 and 4. Taking 300 s and 1000 G cm³ as the arbitrary maximum permissible

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values of T and P, the minimum detectable field in a single reading is about 2×10^{-11} G. In practice such long periods have serious disadvantages (see § 2·10), as do also for most purposes such large values of P.

TABLE 4. APPROXIMATE MINIMUM DIAMETER (CM) OF CIRCULAR MIRROR TO ALLOW THE VALUE OF θ_0 IN TABLE 3 TO BE MEASURED, BASED ON THE EXPERIMENTAL OBSERVATION THAT AN ANGLE OF 2×10^{-6} RADIAN CAN BE DETECTED VISUALLY WITH A 1·0 CM MIRROR

T (sec) \ P (G cm^3)	1000	100	10	1	10^{-1}
10	31	4·6	0·6	0·1	<0·01
30	10	1·5	0·2	0·03	<0·01
100	3·1	0·5	0·06	0·01	<0·01
300	1·0	0·2	0·02	—	<0·01

It will be shown in § 2·10 that a convenient choice would be a period of 60 s and a magnet strength of the order of 100 G cm³, making H_0 about $1{\cdot}4\times10^{-10}$ G. The minimum mirror sizes for this choice of P will be 1·0 cm.

It will be seen from table 4 that small values of T together with large values of P are ruled out owing to the prohibitive size of the mirror required (e.g. the entries above and to the left of the line), unless some means can be found to avoid the limit of angular resolution set by diffraction theory.

The usual method of doing this, by amplification of the spot deflexion by means of some photo-sensitive system (Strong 1938, pp. 326 and 444; Jones 1951, Jones & McCombie 1951), is not directly applicable to the magnetometer. For it will be shown later that, owing to external disturbances, the spot usually drifts much too rapidly to remain centred on the conventional type of fixed photo-sensitive detector. In principle, however, it is possible to use a movable photo-electric detector mounted on a mechanical slide, which can be 'locked on' to the spot and so made to follow it in all its motions by a suitable servo-mechanism. Such 'lock and follow' devices are well known, and if applied to the magnetometer should allow the mirror size to be kept well below the figures in table 4, and so allow the use where suitable of larger values of P and smaller values of T. Such a 'lock and follow' device would have, of course, the great additional advantage of replacing somewhat tiring visual observation of a diffraction image by the much easier observation of the position of the mechanical slide.

I am indebted to Dr H. J. J. Braddick for pointing out that the 'lock and follow' device can be dispensed with if the output of the photo-electric detector, after suitable amplification, be fed back into the coil above the magnetometer used to apply a specified gradient to the magnetometer. In this way the spot is kept locked on to a fixed photo-electric detector, and the magnetic field acting on the magnetometer is measured by the output current of the detector.

2·8·1. *Effect on design of limiting field at specimen*

Suppose the field to be measured H_p arises from a small material specimen with a horizontal magnetic dipole p placed at a distance $z \ll L$ below the lower magnet P of the

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magnetometer with p at right angles to P. We can conveniently define the minimum detectable dipole p_0 as that dipole which gives a field H_0 at P. Thus we have

$$p_0 = z^3 H_0, \tag{20a}$$

or with (9)

$$p_0 = \frac{2\pi\epsilon^{\frac{1}{2}} z^3 I^{\frac{1}{2}}}{TP}. \tag{21}$$

When P is large and z is small, the field $H_1 = P/z^3$ of the magnetometer at the specimen may become so large as to affect the magnetization of the specimen. Thus it may be necessary to limit this field to some assigned maximum value $H_{max.}$, depending on the magnetic properties, in particular the coercivity of the specimen.

Since the couple on the specimen due to the magnetometer equals that on the magnetometer due to the field H_p of the specimen, we have

$$H_{max.}\, p = H_p P. \tag{21a}$$

Taking again the minimum detectable dipole p_0 as that which produces a field H_0 at P, we put $H_p = H_0$ in (21 *a*) and so get

$$p_0 = \frac{H_0 P}{H_{max.}}.$$

Using (9) we obtain

$$p_0 = \frac{2\pi\epsilon^{\frac{1}{2}} I^{\frac{1}{2}}}{TH_{max.}}. \tag{21b}$$

We see that p_0 is inversely proportional to $H_{max.}$ but is independent of P or z. The latter result is understandable since the specimen is supposed always to be brought to that position under the magnetometer where the field has the given value $H_{max.}$. Since now $p_0 \propto I^{\frac{1}{2}}$ but is independent of P, it is advantageous to make the moment of inertia I and so P as small as possible.

The minimum permissible value $P_{min.}$ of P is set by the minimum practicable distance $z_{min.}$ between the specimen and the lower magnets; this distance is governed by, and must be taken to be of the same order as, the size of the specimen. Then we have

$$P_{min.} = H_{max.} z^3_{min.}. \tag{21c}$$

For example, when $H_{max.} = 1$ G and $z_{min.} = 1$ cm, $P_{min.} = 1$ c.g.s.

We see, therefore, that the requirement that the field of the magnetometer at the specimen should not exceed some assigned value leads to a different design of magnetometer compared with the case when the field is unrestricted.

2·8·2. *Design of magnet system when external disturbances are limiting factor*

If for any given suspended system with given period the mean error of a reading due to external disturbances is much larger than that due to the internal Brownian movement, a re-design of the suspended system involving a reduction in I and P so as to make the internal error of the same order as the external error is advantageous, since in this way a higher sensitivity (see equation (5)) can be obtained, and so the observation of the position of the spot is facilitated. No general treatment is possible without some assumption, which with our present knowledge must be largely arbitrary, of the variation of external disturbances with size of magnet system.

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If the special assumption is made that the angular error $\theta_{\text{ext.}}$ due to external causes is independent of the size of the suspended system, then the optimum value I_m of I, that is, the value required to make the internal and external errors equal, is given by putting $\theta_{\text{ext.}} = \theta_0$ in (6*a*), whence

$$I_m = \frac{\epsilon T^2}{4\pi^2\theta^2_{\text{ext.}}}, \tag{21d}$$

from which we see that the larger the external disturbances, the smaller the optimum value of I and so of P.

2·9. *The minimum detectable field with multiple readings*

The limit to the useful decrease of H_0 by increase of T is set by the irregular drift which results from external disturbances which are always present and which tend to increase the errors as the period of observation increases. Only experiments with a particular apparatus can fix the useful limit.

Apart from the disturbances a very high sensitivity could be obtained by using a very long period of swing, say 5 min. Alternatively, one can use fairly small values of T, say 30 s, and take the mean of a considerable number n of readings (see appendix 2). We can compare the error of a single determination of a magnetic field with a given magnetometer used with a very long period T_1, with the error of the mean of n readings of the instrument used with a shorter period T_2. Suppose $nT_2 = T_1$, so that the total duration of the measurements is the same.

From (20) the error of a single reading with the period T will be AT_1^{-1}, where A is a constant. Since for n independent readings the error of the arithmetic mean is proportional to $n^{-\frac{1}{2}}$, the error of the mean of n readings each of duration T_2 will be $An^{-\frac{1}{2}}T_2^{-1}$ or $AT_1^{-1}n^{\frac{1}{2}}$, that is $n^{\frac{1}{2}}$ times the error of a single reading of duration T_1.

This ideal superiority of a single reading of long duration, compared with the mean of a number of readings of short duration taking the same total time, is usually, however, outweighed by the advantage of having a large enough number of readings to allow one to make in each run a reliable estimate of the random errors present, and so to permit one to adopt one of the recognized procedures for discarding any readings subject to unusually large errors, and therefore under suspicion of being influenced by abnormal external disturbances.

2·10. *The required degree of astaticism*

We assume that the two magnets have very nearly but not exactly the same dipole moment. If P' is the resultant moment, the suspended system will respond to a changing *uniform* field as if it were a single magnet of moment P'. If, therefore, during an observation which takes a time T s, the earth's horizontal field at the magnetometer changes by an amount ΔH, then a deflexion will be produced, given by

$$\theta_1 = P'\Delta H/\sigma.$$

It is convenient to introduce the astaticism S of the system, defined as the ratio of the moment of a single magnet to the resultant moment, that is

$$S = P/P'. \tag{22}$$

Then the expression for θ_1 becomes $\quad \theta_1 = P\Delta H/S\sigma. \quad (23)$

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The change ΔH of the local value of the earth's field in the time of observation T can be written

$$\Delta H = T\frac{\mathrm{d}H}{\mathrm{d}t} + \tfrac{1}{2}T^2\frac{\mathrm{d}^2H}{\mathrm{d}t^2} + \dots \tag{24}$$

If the rate of change with time is constant (all terms beyond the first being zero), the effect on the magnetometer will be to make the spot drift at a constant rate across the scale. The effect of such a constant drift is eliminated by taking as the zero position of the spot the mean of its positions before and after the field to be measured has been applied (see § 5·1).

The effect of the second term could, in principle, be eliminated by an analogous method involving the determination of the drift rate before and after the observation (appendix 2). However, at any rate for visual as contrasted with photographic recording, it is found more convenient to use the simple method to eliminate the effect of the first term of (24), leaving the effect of the second to be treated as an error.

If we now introduce $\Delta H = \tfrac{1}{2}T^2\frac{\mathrm{d}^2H}{\mathrm{d}t^2}$ into (23) we get

$$\theta_1 = \tfrac{1}{2}PT^2\frac{\mathrm{d}^2H}{\mathrm{d}t^2}\Big/ S\sigma. \tag{25}$$

Now if this deflexion is not to introduce an appreciable error into the reading, we must ensure that θ_1 is not greater than the average deflexion θ_0 due to the Brownian motion. Putting $\theta_1 = \theta_0$ and using (8) we get

$$S = \tfrac{1}{2}T^2\frac{\mathrm{d}^2H}{\mathrm{d}t^2}\Big/ H_0. \tag{26}$$

This gives the degree of astaticism required to prevent an appreciable error being introduced by a given value of $\mathrm{d}^2H/\mathrm{d}t^2$.

Now inspection of magnetometer records (see § 3·6) shows that on a magnetically very quiet day at a site free from large man-made magnetic disturbances, average values of $\mathrm{d}^2H/\mathrm{d}t^2$ as low as $0{\cdot}1\,\gamma/\mathrm{min}^2$, or about $3\times 10^{-10}\,\mathrm{G\,s^{-2}}$, can be expected. On disturbed days, of course, average values many hundred times higher will be found. Introducing this value into (26) we get

$$S = 1{\cdot}5\times 10^{-10}\,T^2/H_0. \tag{27}$$

Introducing H_0 from (20) we get

$$S = 0{\cdot}0077\,T^3P^{\frac{1}{6}}, \tag{28}$$

whence

$$T = 5{\cdot}8S^{\frac{1}{3}}P^{-\frac{1}{18}}. \tag{28a}$$

I shall show in § 3·5 that I have found it possible to construct magnetometers with an astaticism maintained over long periods as high as 5000. Putting $S = 5000$ in (28) then for P equal to 1, 100 and 1000 G cm^3, we find T equal to 87, 67 and 60 s respectively. These are the maximum values of T which can be allowed for a magnetometer with the specification of § 2·7 (i.e. $G(\beta) = 0{\cdot}009$) and for which $S = 5000$, in order that no appreciable error should be introduced by a field change of $0{\cdot}1\,\gamma/\mathrm{min}^2$. This implies that the lower two rows of table 2 must be excluded.

Now from table 4 we see that the mirror size becomes awkwardly large for the entries above and to the left of the dotted line. Taking these two restrictions together, we see that the acceptable values of T and P are those to the right and above the dotted line in table 2.

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It appears that a magnetometer with $P = 100$ and $T = 67$, giving $H_0 = 1 \cdot 4 \times 10^{-10}$ G, represents roughly the optimum design under the assumed conditions.

In principle the change with time of the earth's uniform field could be prevented from affecting the magnetometer by (*a*) screening the magnetometer with a magnetic screen, or (*b*) detecting the field with some form of detector and compensating the changes by means of suitable Helmholtz coils. It will be shown in the next section that for the experiments with which we shall be mainly concerned, magnetic screening is inapplicable. It will also be shown in § 3·7 that, with present technical methods, field detection and compensation cannot easily reduce the values of d^2H/dt^2 much below that assumed for a quiet day, though it can be of use on disturbed days by bringing down the value nearer that for a quiet day.

2·11. *Consideration of possibility of magnetic shielding*

It is normal practice to surround an astatic galvanometer with a magnetic shield consisting usually of several separate shields, the inner ones being often of Mumetal and the outer of soft iron. With such a shield external magnetic disturbances can be reduced to say 10^{-3} of their unscreened value (see, for instance, Terman 1943, p. 131). However, if such a shield was made large enough to contain the bodies which it was intended to investigate, calculation shows that it would be very heavy and so very difficult to use, and also very expensive. However, a more serious objection exists to the use of magnetic shields in such experiments. Though they attenuate to a very small value an external disturbance, they do not reduce the field inside to zero; in fact, it is usually quite large due to the fact that the Mumetal shields will become magnetized by the earth's field during erection. In fact the use of a magnetic shield appears incompatible with the attainment of a field-free space, unless some practicable system of demagnetization of the complete shield system from inside could be found.

3. Experimental design

3·1. *General arrangement of magnetometer house and apparatus*

The site selected was in the corner of a field 200 yards from the Radio-Astronomical Station at Jodrell Bank, Cheshire.

A copper-nailed wooden building (figure 6), $30 \times 8 \times 8$ ft., was erected on a heavy concrete foundation, with its axis in the magnetic meridian. Internally the walls were lined with a 2 in. expanded ebonite preparation for thermal insulation, with an interior covering of plaster board. The north end of the house was partitioned off by a wall to form a cubical room in which to place the magnetometer.

In the centre of this room a massive concrete block N (figure 7) resting on a 1 in. layer of soft rubber P serves as the base of the stand carrying the magnetometer case. It has a hole down the middle to take the hydraulic cylinder described below. The rubber layer was provided to help to filter out some mechanical vibration, but it is not known how effective it was.

The magnetometer A was suspended by a quartz fibre about 25 cm long and 12 to 18 μ in diameter, chosen, according to the moment of inertia of the magnetometer in use, to give the required period, and was enclosed by a case B with a glass window. Originally the case was of copper, but later was replaced by one of glass. The case was closed at the

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bottom by an aluminium plate to damp the rotational motion of the magnetometer by the eddy currents induced by the lower magnetometer magnets. Vertical adjustments of the quartz fibre holder allowed the damping to be adjusted as required, usually to near critical damping.

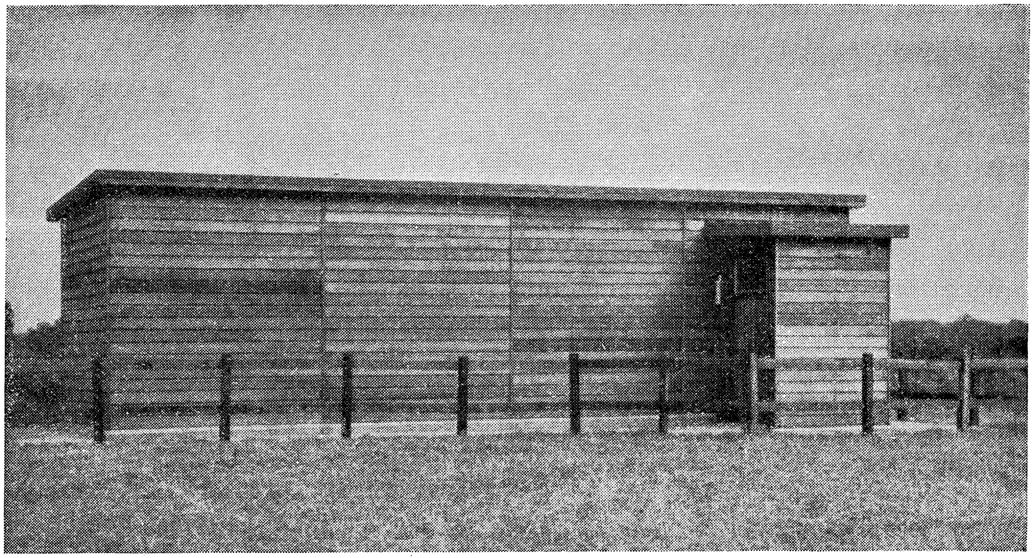

FIGURE 6. Photograph of magnetometer hut.

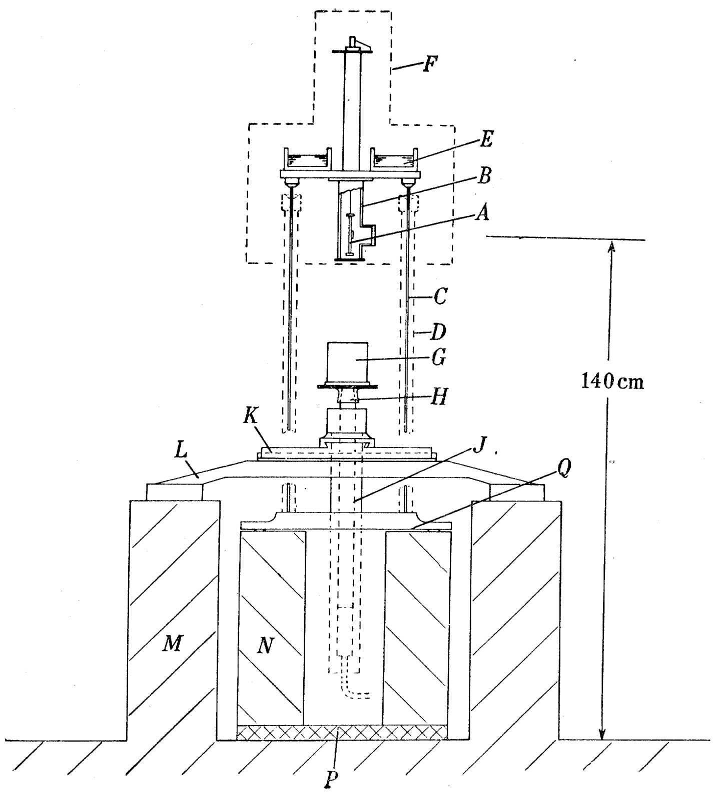

FIGURE 7. Arrangement of magnetometer; description in text.

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The main mechanical filter against external disturbances was provided by an anti-vibration support of the type developed by Müller (1929) and modified by Strong (1938, pp. 328, 591). This was so adjusted as to give the magnetometer case a horizontal period of vibration of about 2 s. In agreement with the results of Müller, such a long period was found to be required in order to filter out earth-borne vibrations, due in this case mainly to railway trains half a mile away, even when the magnetometer had a fairly high degree of dynamic balance. The support consisted of four thin bronze rods *C* in compression, each with an external tubular guard *D* which served the double purpose of shielding the rods from draughts and providing a convenient locking system. The lower ends of the four rods are attached to a heavy bronze base *Q* resting on the concrete block *N*. To achieve the 2 s period found necessary additional mass was provided by lead blocks (not shown). An oil-filled tray *E* for damping the lateral vibration of the whole system was provided.

Recently a new magnetometer of rather lower sensitivity has been set up in a basement room of the Physical laboratories of the University. No anti-vibration device was used but special care was taken to balance dynamically the suspended system. Rather surprisingly the suspended system is subject to very little vibration—much less than the orginal magnetometer at Jodrell Bank—without the anti-vibration device.

An aluminium plate attached to the tubular guards was placed horizontally just under but not touching the bottom of the magnetometer case, and on this rested a wooden box *F*, shown dotted, which acted as a draught and thermal shield.

In order to make a measurement it is necessary to be able to produce the effect to be measured in a time short compared with the period *T* of the instrument. This is achieved by mounting the specimen *G*, whose magnetic properties are to be measured, on a vertical hydraulic piston *H*, so that the specimen can quickly and smoothly be moved up to and away from a position just under the magnetometer. The top of the piston rod is coned, and on this cone, which is greased, fits a Dural head to which is attached a Perspex tray on which the specimen is placed. The greased cone allows the specimen to be easily rotated in azimuth by an endless string worked from the outer room. A hydraulic ram fitted with a hand-worked lever and placed at the far end of the hut served to actuate the piston. Motor-car brake fluid was used as a liquid owing to its suitable viscous properties. The cylinder *J* in which the piston works was fitted to a two-co-ordinate traverse mechanism *K* to allow accurate centring under the magnetometer. The traverse mechanism was made like a lathe slide rest but in non-magnetic bronze. It was carried on a bronze bridge *L*, resting on two concrete blocks *M* which were integral with the concrete base of the hut. By these means a heavy mass, e.g. up to 30 kg, could be traversed to any desired position under the magnetometer (accuracy 0·1 mm) and could be moved up and down some 20 cm in 5 s or less, without any appreciable vibration being communicated to the magnetometer.

All metal parts were initially made of electrolytic copper, commercial non-magnetic bronze or of Dural. However, it was found that these metals, though the best available, were apt to be too magnetic to be used with safety in the immediate neighbourhood of the suspended system. Tests with the magnetometer itself showed that various non-metallic substances, especially glass, wood and Perspex, are generally much freer from ferromagnetic impurities than metal. As a result, the cylindrical copper magnetometer case was later replaced by a cylinder of glass closed at the bottom by an aluminium damping plate. In

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still later designs, commercial laminated wood has been found very suitable for the magnetometer case.

At the south end of the hut and 5 m from the magnetometer, a concrete pillar carried a lamp fitted with spectroscopic slit and a millimetre scale, or a low-power microscope. Observations were normally made visually by eye or with a low-power microscope. Photographic recording was tried but found to be not much more accurate, and much more laborious, than direct visual observation.

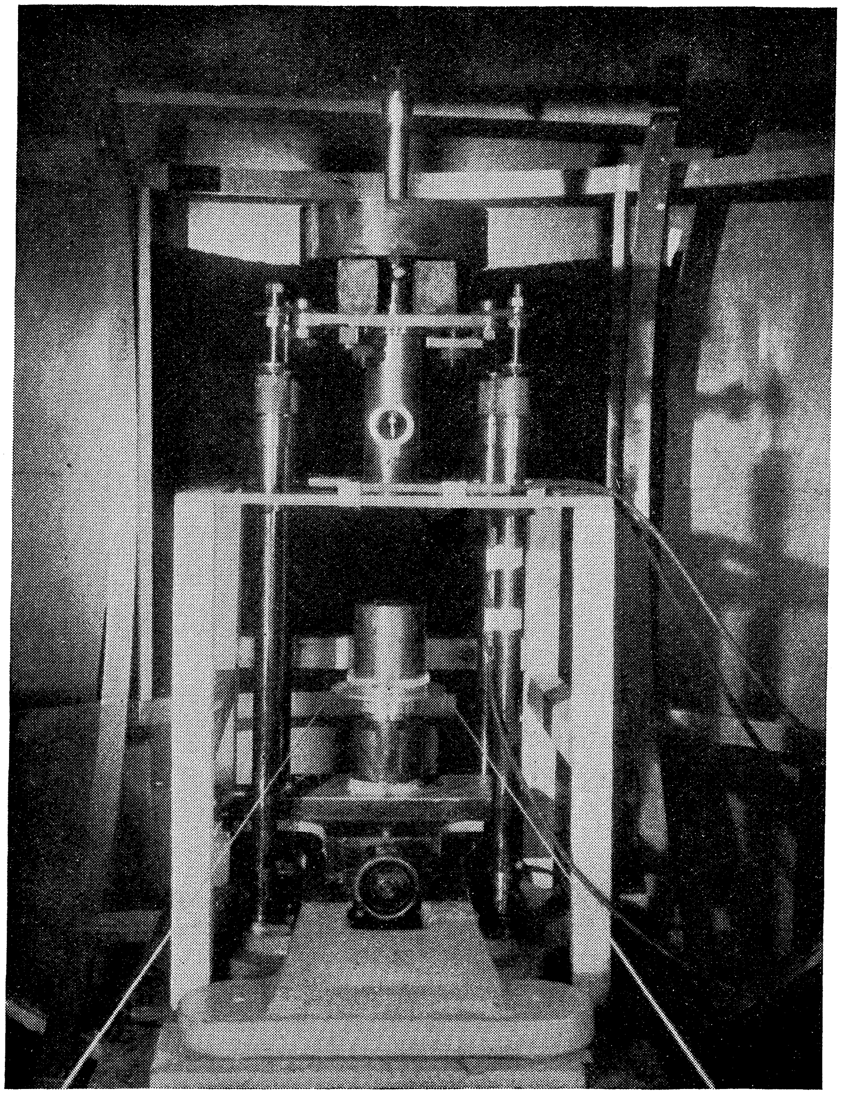

FIGURE 8. Photograph showing magnetometer case, Julius suspension traversing mechanism and piston carrying 10 cm gold cylinder.

Figure 8 shows a photograph of the complete magnetometer with the wooden cover removed. Most of the parts showing in the photograph can be identified by reference to figure 7. Not shown in the diagram but to be seen in the photograph are the following: one handle of the lathe-bed traversing mechanism; the strings used to rotate the specimen; two lead blocks under the oil-filled damping tray; a wooden frame surrounding the

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supporting pillar and carrying the fluxgate detector described in § 3·6; parts of the E and V compensating coils.

The magnetometer room was kept within $\pm 0{\cdot}5°$ C of a constant temperature by means of a commercial thermostatic element, which controlled a 400 W heating system consisting of eight underrun 150 W lamp bulbs disposed around the periphery of the floor. In the later experiments a small commercial squirrel cage fan in one corner of the room served to circulate the air slowly, and so to keep the temperature of the room uniform. Usually the spot position varied by 10 to 100 mm for 1° C change in room temperature. This rapid variation resulted from a considerable degree of magnetic control due to magnetic impurities in the constructional materials of the magnetometer case. The large magnitude of the effect was somewhat surprising, but was probably due to the fact that the equilibrium position of the anti-vibration suspension is very sensitive to temperature. A shift of this position alters the spatial relation of the magnetometer itself to the slightly magnetic parts of the case surrounding it, and so alters the zero position of the spot.

3·2. *Field compensating coils*

In order to be able to carry out experiments in a nearly field-free space, three pairs of Helmholtz coils were placed around the magnetometer to compensate the north-south component H of the earth's field, the vertical component V and the east-west component E. The radii of the three pairs of coils to compensate H, V and E are 115, 107 and 100 cm respectively; each coil had two windings of 160 turns each, a main winding of 20 s.w.g. copper wire and a subsidiary winding of 30 s.w.g. The fields at the centre of the three coil systems were, respectively, 1·25, 1·35 and 1·45 G/A.

Thus a current of about 140 mA in the H coils compensated the earth's local horizontal component of 0·18 G, and a current of 340 mA the vertical component of 0·45 G. The E coils were set about $\frac{1}{2}°$ out of the meridian, so that a current of about 1 mA was required to compensate the small east-west field.

The subsidiary windings were used for two different purposes. By passing small currents of the order of a few milliamperes through them, known magnetic fields of a few milligauss could be applied to the magnetometer to test its degree of astaticism; or the coils could be used to compensate automatically small changes in the earth's field by means of a 'Fluxgate' detector, which will be described in § 3·5.

The field produced by a Helmholtz coil system at a point near the centre differs from that at the centre by terms proportional to the fourth power of the distance from the centre. From the expression quoted by Chapman & Bartels (1940) one finds that if the earth's field is exactly compensated at the centre, the lack of compensation for the coils used will be less than 1 γ at a distance of 10 cm. Thus a space of dimensions about $20 \times 20 \times 20$ cm, in which the field is everywhere less than 1 γ, can be achieved by such a coil system.

3·3. *The magnetometers*

Various types have been constructed and used. The main parameters are given in table 5. The symbols have the same meaning as in § 2. The first nine rows give constants of the magnetometer itself. The figures in the last five rows depend on the quartz fibre used. The quantity δ_0 is the r.m.s. deflexion expected from the Brownian movement, and is given

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by $\delta_0 = 10^7\theta_0$ microns, since the scale distance is 5 m. The values of H_0 are the magnetic fields which must act on one magnet to give a deflexion equal to δ_0. All the magnetometers are fitted with concave mirrors with 5 m radius of curvature. The mirrors were 1·0 cm in diameter for nos. 1 and 2 and 0·5 cm for no. 3.

Table 5. Details of the magnetometers

According to equation (7) the quantity $P/I^{\frac{1}{2}}$ is proportional to the signal-noise ratio for given T

	1	2	3
material of magnets	Vectolite	Caslox	Alcomax IV
total mass, M (g)	4·6	12·0	1·02
moment of inertia, I	0·165	0·60	0·041
moment of inertia of magnet, I_0	0·037	0·114	0·010
$\alpha = I/I_0$	4·5	5·3	4·1
dipole moment, P (c.g.s.)	60	54	15·0
separation of magnets, l (cm)	3·8	4·5	14·8
quadrupole moment, $Q = Pl$	230	240	225
$P/I^{\frac{1}{2}}$	140	69	75
length/breadth ratio, β	0·16	0·2	4·5
period, T (sec)	25	45	30
thermal deflexion, δ_0 (μ)	24	15	45
gradient for 1 mm deflexion g (G/cm)	$3·1 \times 10^{-9}$	$6·7 \times 10^{-9}$	$1·0 \times 10^{-9}$
field for 1 mm deflexion $g' = gl$, (G)	$1·2 \times 10^{-8}$	$3·0 \times 10^{-8}$	$1·5 \times 10^{-8}$
minimum detectable field, H_0	$2·9 \times 10^{-10}$	$4·5 \times 10^{-10}$	$6·7 \times 10^{-10}$

Some constructional details of the suspended magnetometer system are shown in figure 9. A fairly high degree of dynamic balance was necessary to avoid undue coupling between the pendulous and the rotational oscillation of the system.

The Vectolite magnetometer was the first to be constructed. In accordance with the argument of § 2·4 two specimens of Vectolite *AA* of square cross-section, 3·2 cm long by 0·5 cm broad, giving a fineness ratio of 0·16, were cut and magnetized transversely. They were attached together by cementing them to a thin aluminium sheet former *B*, which was itself attached to a Perspex rod *C*, which carried the mirror *D* and the two pairs of trimming magnets *EE*, *EE*, which were used in the way described in the next section to bring the whole suspended system to a high degree of astaticism. The small trimming magnets were attached to the heads of small screws inserted in tapped holes. The magnets could then be rotated to any desired angle.

The magnetic moment of each Vectolite magnet was 60 c.g.s.; the stronger pair of trimming magnets with which coarse adjustment was done had moments about 1·0 c.g.s., and the weaker pair used for fine adjustment about 0·2 c.g.s.

This magnetometer was found very satisfactory and showed the great effectiveness of this method of achieving a high astaticism. Under favourable conditions the mean error of a reading was not very much larger than the calculated thermal value.

It was thought possible that the fact that the mean error was normally considerably greater than the thermal error might be due to fluctuations of the vertical gradient of the earth's horizontal field. To eliminate this effect a parastatic system was built, with four magnets consisting of pieces of Caslox 2·5 × 0·55 cm magnetized transversely. As this material is machinable, holes were drilled along their long axis, and they were threaded like beads on to lengths of phosphor-bronze wire which were attached to a short Perspex

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rod carrying two double-sided concave mirrors. The two inner magnets pointed in the same direction and the two outer ones in the opposite direction. Such a system is not deflected by a field with a constant vertical gradient. The use of two mirrors, D_1, D_2 each with both sides concave, allowed the magnetometer to be used in each of four azimuths. In the static body experiment described in § 6, measurements could then be made with the lower magnet dipole oriented north, south, east and west. By suitably averaging the results it was therefore possible to eliminate any residual effect due to the magnetic interaction of

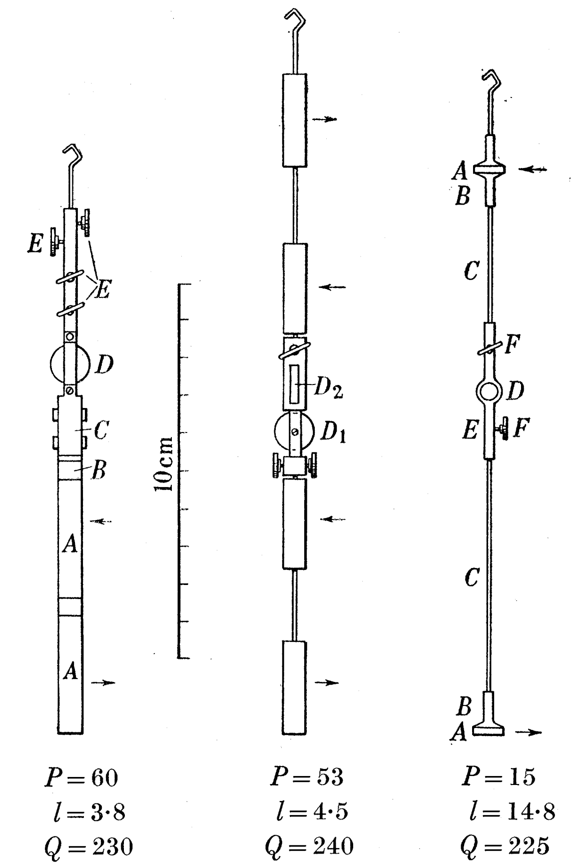

FIGURE 9. Details of three magnetometers; description in text.

the magnet with the specimen. This magnetometer also proved very satisfactory, but the errors of the readings were no lower than with the Vectolite magnetometer. It was therefore concluded that these errors were not due to changes in the vertical gradient, and that there were no advantages in using a parastatic system.

The third magnetometer was made with weaker magnets ($P = 15$) in order to reduce the field of the magnetometer at the specimen, and had a greater distance between the magnets ($L = 14{\cdot}8$ cm) so as to obtain as large as possible a difference between the field at the two magnets produced by the specimens under investigation.

The magnets were of Alcomax IV, $8{\cdot}0 \times 1{\cdot}8 \times 1{\cdot}8$ mm, magnetized longitudinally. The fineness ratio of 4·5 was chosen to be near the optimum for a longitudinal magnet (figure 5).

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The magnets AA were cemented to Dural connecting pieces BB, into which were cemented glass rods CC, 0·5 mm in diameter, the other ends of which were cemented to a Dural rod E carrying a mirror D, 5 mm in diameter, and the two trimming magnets FF.

This magnetometer was used for the final set of measurements in the static-body experiment and is still in use. With it an extensive set of measurements of weakly magnetized substances is being made.

In general one can conclude that there are many methods of construction which work equally well. The important factors to achieve are a high value of $P/I^{\frac{1}{2}}$ and a high degree of mechanical stability, which will permit a high degree of astaticism to be maintained. A high degree of dynamic balancing is essential to minimize the effect on the spot of external mechanical disturbances.

3·4. *Measurement of the sensitivity*

The sensitivity of a magnetometer is measured by applying a non-uniform horizontal magnetic field by means of a small coil placed on the roof of the hut vertically above the magnetometer, and at a distance from the magnetometer much greater than the distance L between the magnets. Thus the field will have an appreciably constant vertical gradient $\mathrm{d}H/\mathrm{d}z$ at the magnetometer, and so will produce a difference of magnetic intensity $\Delta H = L(\mathrm{d}H/\mathrm{d}z)$ on the two magnets. The *gradient sensitivity*, as measured in this way, will be denoted by g, and is conveniently expressed in the form '1 mm scale deflexion corresponds to g gauss/cm'. It is often useful to express the gradient sensitivity in terms of the difference of field ΔH at the two magnets, rather than in terms of the gradient. Thus we can also express the gradient sensitivity by the statement that '1 mm deflexion corresponds to g' gauss', where

$$g' = gL. \tag{29}$$

Strictly the quantities g and g' should be called the reciprocal sensitivities.

From equation (5) it is seen that the deflexion of given magnetometers used on different fibres is proportional to T^2. Thus the quantities g and g' are inversely proportional to T^2.

Actual values of g and g' for the three magnetometers at the given periods T are given in the last two rows of table 7.

In order to calculate the standard error $\delta_0 = 2R\theta_0$, where R is the distance of the scale (5 m), due to the thermal oscillation of the suspended system, we need to know either the torsional control constant σ using equation (6), or the free period T and the moment of inertia I using (6 a). Now both σ and T are often affected markedly by the presence of some degree of magnetic control superimposed on the elastic control of the quartz fibre, and may change considerably from time to time. Moreover, the free period T is not measurable directly when the magnetometer is in its working condition of near critical damping. It is therefore often convenient to express δ_0 in terms of the directly measured quantity g defined above as the field gradient to give 1 mm deflexion. It is easily shown that

$$\delta_0 = \left(\frac{2R\epsilon}{LPg}\right)^{\frac{1}{2}}, \tag{30}$$

where δ and R are measured in millimetres, and as before P is the dipole moment of one of the magnets and L is their distance apart. This expression has been used to calculate the value of δ_0 given in table 7.

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3·5. *Method of astaticizing the magnetometers*

The two magnets were made as nearly as possible equal in strength before mounting on their holder. With Vectolite and Caslox magnets the final adjustment was done by filing or rubbing with emery paper. The Alcomax magnets were ground on a grinding machine as nearly as possible equal in size, and the final adjustment to near equality of dipole strength made by partial demagnetization of the stronger by an a.c. field, as described by Johnson & Steiner (1937). A slight degree of demagnetization was applied to both magnets in order to improve their stability.

When mounted, a first test of the astaticism S was obtained by timing the magnetometer period in the earth's field (assumed uniform). If T_1 is the period of swing with the magnets mounted nearly anti-parallel, and if T_2 is the period when mounted parallel, then it is easily seen that

$$S \equiv \frac{P}{P'} = \frac{1}{2}\frac{T_1^2}{T_2^2}, \tag{30a}$$

when, as before, P' is the residual out-of-balance moment. It is usually found that values of S up to 100, but not easily much higher, can be obtained in this way.

To reach much higher values, the two trimming magnets of dipole moment p_1 and p_2, somewhat greater than the two horizontal components of the out-of-balance moment P', were rotated so as to compensate as nearly as possible the out-of-balance moment of the two main magnets.

If the trimming magnets make angles ψ_1 and ψ_2 with the vertical axis of the magnetometer, they will introduce horizontal components $p_1 \sin \psi_1$ and $p_2 \sin \psi_2$, which, by suitable [redacted]

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These expressions allow the component astaticisms to be calculated from the directly measured gradient and uniform field sensitivities, and the distance L between the magnets.

To alter the angle of a trimming magnet it is necessary with the present apparatus to unhook the magnetometer from its fibre. A new magnetometer has been designed and made, in which a mechanism is incorporated whereby the magnet system itself can be locked in its case and the adjustment of the trimming magnets can be done by means of external adjusting knobs.

The trimming magnets are preferably made of small lengths of wire of good magnetic properties. A convenient material is Vicalloy which is ductile and so can be obtained in the form of wires.

Owing to temperature effects, ageing of the magnets, etc., the astaticism does not remain quite constant over a long period of time. However, the constancy in most cases is adequate for the present experiment. An astaticism between 3000 and 5000 could be maintained for weeks on end. Considerably higher values could be maintained for short periods. There is, moreover, an upper limit to the actual astaticism which can be attained, owing to a limitation on the method of measuring it. As explained above, the measurement is made by applying a *uniform* field to the magnetometer by passing a current through the compensating coils. Now, owing to geometrical imperfections in the shape of the coils, the field at their centre is not quite uniform, and this small non-uniformity introduces an error into the field sensitivity and so to the astaticism. I estimate that with the present coils an astaticism of about 10000 is probably the highest which can be measured reliably.

3·6. *The fluxgate system*

For some experiments, for instance, for both the static and rotating body experiments, it is essential to bring the field at the centre of the Helmholtz coil system as nearly as possible to zero. The method adopted was the use of the 'Fluxgate' system of measuring weak fields, used first by Aschenbrenner & Goubau (1936) (see Chapman & Bartels 1940, p. 59), and developed during the war in the U.S.A. for the detection of submarines from the air and for aerial magnetic surveying. A detecting element consists in principle of a thin Mumetal wire surrounded by a coil carrying an alternative current of frequency ν. In practice a detecting head consists of two Mumetal strips placed parallel to each other in a single solenoid producing the driving a.c. field. Each Mumetal wire is wound with its own pick-up coil, and then are connected in opposite senses to the two arms of an a.c. bridge. Owing to the non-linear magnetization characteristic of the Mumetal wire, the e.m.f. induced in a pick-up coil surrounding the wire contains higher harmonics of the driving frequency ν.

Analysis shows that the amplitudes of the even harmonics are proportional to the absolute value of the steady field acting on the wire. By means of suitable filters and amplifiers an output meter can, therefore, be made to record the steady field at the detector. Such an instrument can either be of the second harmonic type, in which only this harmonic is used, or it can be of the 'peak' type, in which a wide range of even harmonics are utilized. The ultimate noise level of such an instrument with a recording time of about 0·5 s appears to be about 2×10^{-7} G ($0{\cdot}02\,\gamma$) (Goldsack 1949, unpublished work), though in the instrument described below the noise level was about $0{\cdot}1\,\gamma$.

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If such a detecting element is placed at the centre of the Helmholtz coil system, the output meter will record the residual field and so allow the current in the coils to be adjusted so as to make this residual field nearly zero. Moreover, if the output current is passed in the right sense through the subsidiary windings of the Helmholtz coils, it can be made to compensate by negative feedback the residual field at the detector, and so automatically bring this field nearly to zero.

Three identical fluxgate instruments of the 'peak' type were designed and made for me by the Research Laboratories of Elliott Brothers (London). A description of the instrument has been given in a paper by Brewer, Squires & Ross (1951), in which will also be found reference to previously published papers and patent specifications. In each instrument a change of $0{\cdot}1\,\gamma$ at the detector head produces an output current of $0{\cdot}5$ mA with feedback, with a time of response of about $0{\cdot}5$ s. The degree of compensation achieved by passing this output current through the Helmholtz coils can be calculated as for other negative feedback systems, as follows (Goldsack, unpublished work).

Suppose (*a*) that a current of 1 A in the coils produces a field of H gauss at their centre, and (*b*) that the amplifier system is such that an output current A amperes is produced by a change of field at the detector of 1 G.

If now the external field (e.g. the earth's field) changes by δH_0 gauss, current from the amplifier will flow through the feedback coils in such a direction as to oppose this change, and will reduce its magnitude to a value δH given by

$$\delta H_0 - HA\delta H = \delta H,$$

which can be written

$$\frac{\delta H}{\delta H_0} = \frac{1}{1+HA} \sim \frac{1}{HA}, \tag{32}$$

since $HA \gg 1$.

For our feedback coils H had the value of about $1{\cdot}2$ G/A, and for the Elliott instrument A had the value of $0{\cdot}5\,\text{mA}/0{\cdot}1\,\gamma$ or 500 A/G. Thus the product HA had the value 600 A/G. We see therefore that such a system compensates changes in the external field to an accuracy of 1 in 600. The output current due to a change δH_0 in the external field is $A\delta H$, and from (32) this equals $\delta H_0/H$. Thus the output current is directly proportional to the change of field, and is governed only by the constants of the feedback coil and not at all by the characteristics of the amplifier.

The range of field compensation is set by the output stage of the amplifier and in our instrument is about $\pm 500\,\gamma$; however, in practice, the instrument is only used to compensate fields up to $\pm 100\,\gamma$. Within this range variations of external field are reduced to $\frac{1}{600}$ of their value that is to below $\frac{100}{600} \sim 0{\cdot}2\,\gamma$.

The instrument had three separate detecting heads and amplifying systems so as to enable all three components of the field in the Helmholtz coils to be compensated. The east-west fluxgate detector head can be seen in figure 8 attached to a wooden frame surrounding the magnetometer support.

This instrument not only maintains the three components of the field at the centre of the coils within $0{\cdot}2\,\gamma$ of zero, but the reading of an output meter gives the variation of the external field at the detecting head.

Using a pen recorder, the variation of the earth's field together with the local man-made disturbances can be conveniently recorded. This facility is of great use in that movements

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of the magnetometer spot can be correlated with changes in the external magnetic field. Magnetically quiet periods can be chosen when accurate measurements are required. In figure 10 are shown some typical reproductions of records of the east-west components of the earth's field in quiet, moderately disturbed and very disturbed conditions.

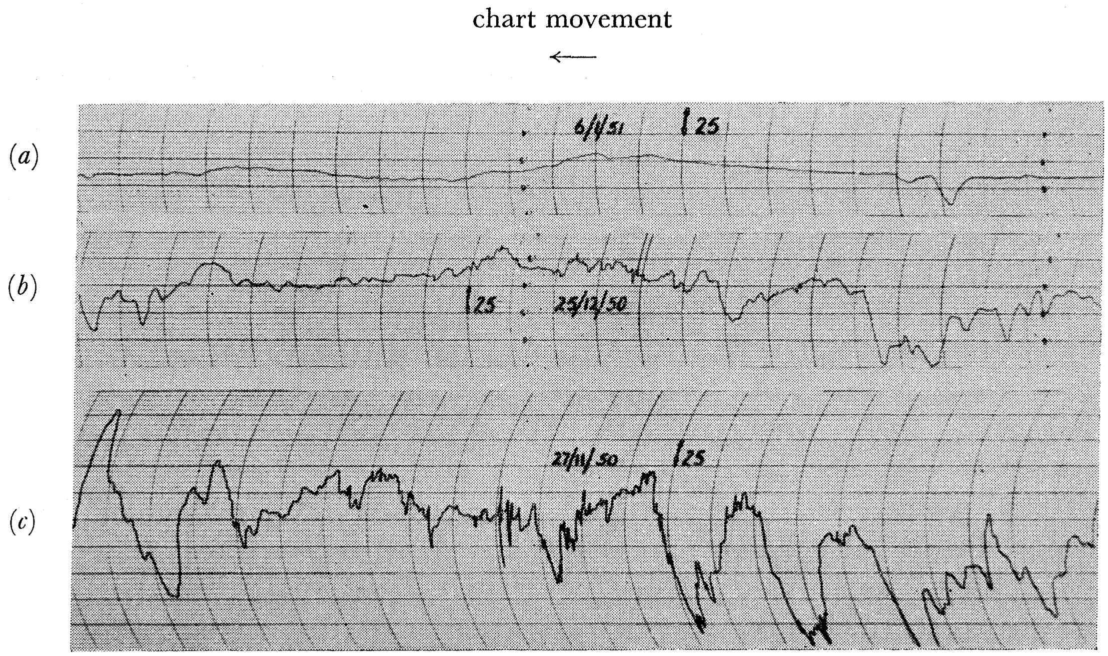

Figure 10. Records of east-west fluxgate for three complete days, when the magnetic conditions were (*a*) quiet, (*b*) moderately disturbed, (*c*) very disturbed. The chart speed was 1 division in 1 h. One large division of the vertical scale represents 25 γ.

A direct test of the effectiveness of the field nulling system provided by the fluxgate detector and feedback coils is obtained by recording the position of the magnetometer spot when the current in the H compensating coil is altered. Figure 11 shows a typical record. Over the range of current 138 to 146 mA, corresponding to a change of field at the centre of the coil of about 1000 γ, the spot remains almost unaffected, showing that the current from the fluxgate amplifier through the subsidiary windings of the compensating coils almost exactly cancels the change of field due to the change of current in the main windings. Outside this current range the amplifier saturates and no more compensation occurs; so the spot is deflected by the change of field according to the astaticism of the magnetometer.

The ratio of the slopes of the steep and the flat portion of such a curve gives, in principle, the compensation factor; the observed ratio was not inconsistent with the designed factor of 600, but no attempt was made to make an actual experimental determination of the factor.

3·7. *Effect of changes of the earth's field on the magnetometer*

An inspection of the fluxgate records shows that under very quiet magnetic conditions the field may vary by as little as 0·1 γ/min. Since a reading takes a time of about 1 min and the fluxgate only compensates to 0·1 γ, the use of the fluxgate field compensation cannot reduce the field variation below 0·1 γ/min, and so is of no value under very quiet conditions.

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Consider the Alcomax magnetometer working at a gradient sensitivity g' of 1 mm to 10^{-8} G and at an astaticism of 3000. Then from (31) the field sensitivity f is 1 mm to $3000 \times 10^{-8} G = 3\gamma$. Thus a rate of change of field of 0·1 γ/min will produce a drift of the spot of $\frac{1}{30}$ mm/min. On a normal day field changes of say 3 γ/min are common and so will produce a drift of 1 mm/min. This is about the fastest drift which allows reasonably precise

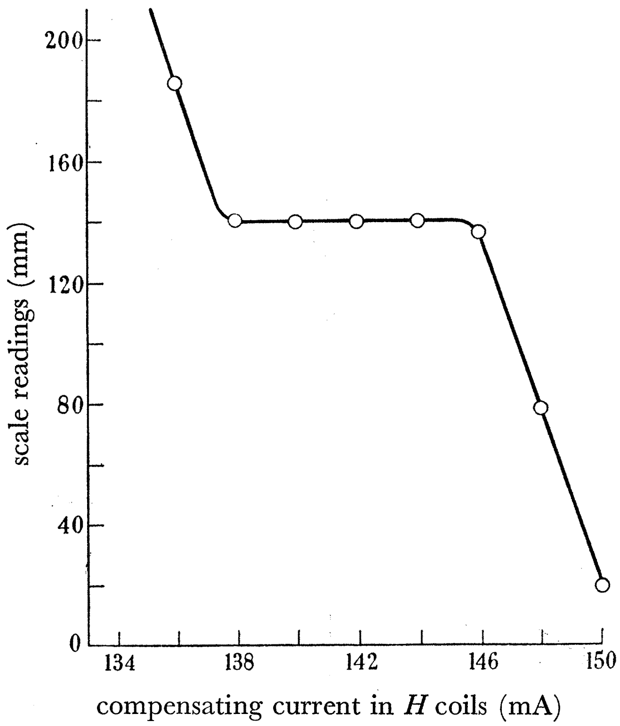

FIGURE 11. Effect of fluxgate compensation. Position of magnetometer spot plotted against current in H coils. The astaticism of the magnetometer was 1500.

measurements with the magnetometer. Under these conditions the fluxgate compensation should markedly reduce the drift due to changes of the earth's uniform field. However, as shown in §§ 2·9 and 5·1, the error of a calculated deflexion is independent of a constant drift, but depends on the second time derivative d^2H/dt^2. The fluxgate records show that under very quiet conditions values as low as 0·1 γ/min^2 are found. This is the value assumed in § 2·10 to estimate the required degree of astaticism. Under normal conditions values ten or a hundred times as high are common.

4. THE THEORY OF THE MEASUREMENT OF THE MAGNETISM OF WEAKLY MAGNETIZED SPECIMENS

4·1. *Permanent magnetism of small specimens*

If the size of the specimen is small compared with its distance z from the lower magnet of the magnetometer, the magnetic field of the specimen at the magnetometer can be represented by the field of a point dipole. For simplicity we shall assume that the distance L between the magnets of the magnetometer is large compared with z, so that only the lower magnet need be considered.

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Consider first a small magnet *on the axis of the magnetometer* and at a distance z below the lower magnet P_1. The plane containing P_1 is defined as the zOy plane (figure 12). If the dipole p of the magnet makes an angle of ϕ with the horizontal, its horizontal component will be $p_x = p\cos\phi$, and the horizontal magnetic field at P_1 in the Ox direction will be

$$H_x = (p_x/z^3)\cos\chi, \tag{33}$$

where χ is the angle with Ox made by the projection of p on to the xOy plane.

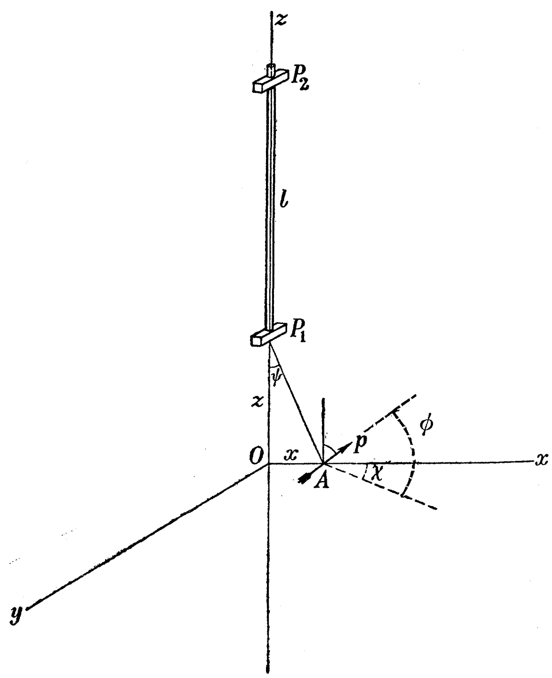

FIGURE 12. Small magnet, dipole p near lower magnet P_1 of magnetometer.

If the magnet p is rotated in azimuth around the vertical axis, the deflecting field at P_1 will be a simple harmonic function of azimuth. From the amplitude of the measured field, together with the value of z and the measured sensitivity of the magnetometer, the value of p_x is determined.

To obtain the vertical component $p_z = p\sin\phi$ the specimen is traversed horizontally a distance x at right angles to the plane of the magnetometer (figure 12). The vertical component p_z will produce a horizontal field at P_1 of magnitude

$$H_z = \frac{3p_z}{2z^3}\cos^3\psi\sin 2\psi, \tag{34}$$

where $x/z = \tan\psi$. For small values of ψ this becomes

$$H_z = 3p_z x/z^4. \tag{35}$$

We see therefore that the deflexion of the magnetometer is proportional to the displacement x. The slope of the curve of H_z plotted against x thus allows p_z to be calculated. From the values of p_x and p_z, p can be directly determined and ϕ can be calculated from the expression

$$\tan\phi = \frac{p_z}{p_x} = \frac{H_z z}{H_x 3x}. \tag{36}$$

These relations can be tested experimentally by using a small test magnet (see § 5·6).

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4·2. *Permanent magnetism of large specimens*

When the specimen, though weakly magnetized, is comparable in size with the distance to the magnetometer, it is preferable when possible to make it of some regular geometrical shape for which exact calculations are possible. A convenient one is a circular cylinder with its axis vertical. I am indebted to Dr A. Papapetrou for calculating approximately the field outside and near the axis of a circular cylinder of radius a and height h, magnetized uniformly to a strength γ_z per unit volume vertically and γ_x horizontally.

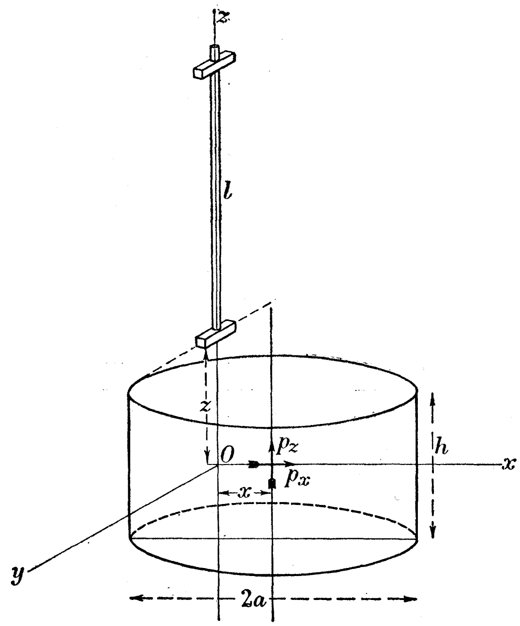

FIGURE 13. Uniformly magnetized cylinder near lower magnet of magnetometer.

Consider the field at point z above the *centre* of the cylinder at a distance x off the axis at right angles to the plane of the magnetometer (figure 13). Denoting the total horizontal and vertical dipoles of the specimen by p_x and p_z we have

$$\left.\begin{aligned} P_x &= \pi a^2 h \gamma_x, \\ P_z &= \pi a^2 h \gamma_z. \end{aligned}\right\} \tag{37}$$

If we write

$$\left.\begin{aligned} u &= h/2z, \\ v &= a/z, \end{aligned}\right\} \tag{38}$$

then Papapetrou's expressions for the horizontal field due to the horizontal and vertical dipoles are

$$H_x = P_x F_x(u, v)/z^3, \tag{39}$$

where

$$F_x = \frac{1}{2uv^2}\left[\frac{1-u}{\{(1-u)^2+v^2\}^{\frac{1}{2}}} - \frac{1+u}{\{(1+u)^2+v^2\}^{\frac{1}{2}}}\right] \tag{40}$$

and

$$H_z = 3P_z x F_z(u, v)/z^4 + \dots, \tag{41}$$

where

$$F_z = \frac{1}{6u}\left[\frac{1}{\{(1-u)^2+v^2\}^{\frac{3}{2}}} - \frac{1}{\{(1+u)^2+v^2\}^{\frac{3}{2}}}\right]. \tag{42}$$

The calculations are based on the assumption that $x \ll [u^2+(z-\frac{1}{2}h)^2]^{\frac{1}{2}}$. The field is developed in a power series in x and y, and only the terms which are linear in x are considered.

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Again we have for the angle ϕ of the total moment with the horizontal

$$\tan\phi = \frac{P_z}{P_x} = \frac{H_z z}{H_x 3x}\frac{F_x(u,v)}{F_z(u,v)}. \tag{43}$$

Comparing these expressions with (33), (34) and (35), we see that the functions F_x and F_z represent the factors by which the field of the finite cylinder differs from that of a point dipole at its centre. It follows therefore that for values of z large compared with a and h, that is, when u and v are both markedly less than unity, F_x and F_z must tend to unity. In table 6 are given some values of these functions for different values of u and v. It will be noted that $u = 1$ corresponds to $z = \frac{1}{2}h$, that is, to a point on the upper surface of the cylinder.

TABLE 6. FIELD OF UNIFORMLY MAGNETIZED CYLINDER. VALUES OF $F_x(u, v)$ $F_z(u, v)$ AND F_x/F_z FOR DIFFERENT VALUES OF u AND v CALCULATED FROM EQUATIONS (40) AND (42)

$u = h/2z$; $v = a/z$; $u/v = h/2a$ and equals ratio of height to diameter of cylinder

z/a	$h=2a$ $(v=u)$				$h=a$ $(v=2u)$			
$(=1/v)$	u	F_z	F_x	F_x/F_z	u	F_z	F_x	F_x/F_z
0·1	—	—	—	—	—	—	—	—
0·2	—	—	—	—	—	—	—	—
0·5	—	—	—	—	1·0	0·0135	0·0884	6·53
1·0	1·00	0·152	0·477	2·94	0·5	0·182	0·385	2·11
1·5	0·667	0·535	0·815	1·51	0·333	0·440	0·617	1·40
2·0	0·50	0·855	0·965	1·28	0·250	0·632	0·763	1·20
2·5	0·40	0·935	1·01	1·08	0·20	0·753	0·848	1·13
3·0	0·333	1·02	1·02	1·00	0·167	0·838	0·890	1·06
4·0	0·20	1·02	1·02	1·00	0·10	0·931	0·940	1·01
5·0	0·25	1·02	1·02	1·00	0·125	0·967	0·968	1·00
7·5	0·133	1·01	1·01	1·00	0·067	0·980	0·981	1·00
10·0	0·10	1·01	1·01	1·00	0·05	0·989	0·990	1·00
15·0	0·067	1·00	1·00	1·00	0·033	0·999	1·00	1·00
20·0	0·05	1·00	1·00	1·00	0·025	1·00	1·00	1·00

z/a	$h=\frac{2}{5}a$ $(v=5u)$				$h=\frac{1}{5}a$ $(v=10u)$			
$(=1/v)$	u	F_x	F_z	F_x/F_z	u	F_x	F_z	F_x/F_z
0·1	—	—	—	—	1·00	0·00001	0·00098	—
0·2	1·00	0·00026	0·00744	28·6	0·50	0·00028	0·0075	27·0
0·5	0·40	0·0177	0·0894	5·05	0·20	0·0177	0·0894	5·05
1·0	0·25	0·178	0·359	2·01	0·10	0·177	0·355	2·01
1·5	0·1333	0·400	0·555	1·41	0·0667	0·398	0·553	1·41
2·0	0·100	0·582	0·725	1·15	0·050	0·590	0·723	1·15
2·5	0·080	0·696	0·812	1·10	0·040	0·694	0·808	1·10
3·0	0·0667	0·774	0·862	1·06	0·0333	0·772	0·859	1·06
4·0	0·040	0·865	0·912	1·04	0·020	0·863	0·910	1·04
5·0	0·050	0·908	0·947	1·02	0·025	0·907	0·945	1·02
7·5	0·00267	0·948	0·974	1·02	0·0133	0·947	0·973	1·02
10·0	0·020	0·976	0·986	1·01	0·010	0·976	0·985	1·01
15·0	0·0137	0·989	0·994	1·00	0·0067	0·989	0·994	1·00
20·0	0·010	0·994	0·996	1·00	0·005	0·994	0·996	1·00

In the foregoing calculations the horizontal component of the magnetization has been assumed to be at right angles to the magnetometer magnet P_1. When the specimen is rotated so that the horizontal component makes an angle χ with the x-axis, the resultant field due to the horizontal component deflecting the magnetometer will be $H_x \cos\chi$, but the field due to the vertical component will remain equal to H_z.

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By measuring H_x, H_z, x and z, and using the relevant values of F_x/F_z, the inclination ϕ of the magnetization with the horizontal can be calculated.

The application of these expressions to the precise determination of the azimuth and inclination of the permanent magnetic moments of weakly magnetized geological specimens will be described in a subsequent paper. The method is particularly valuable when it is necessary to measure the inclination of the magnetization of thin cylindrical specimens.

4·3. *Measurement of the magnetic susceptibility of specimens*

Normally the currents in the three-component Helmholtz coils are adjusted so as to make the field at their centre nearly zero. Thus, for instance, measurements of the permanent magnetism of a specimen are made by rotating and traversing the specimen near the magnetometer with the external field nearly zero. If, however, the external field is made different from zero, then the specimen will acquire an induced magnetization which can be measured.

Suppose that the plane of the magnetometer is east-west, i.e. in the plane $z\,Oy$ (figure 13), and that the north-south component of the external field at the specimen is altered from zero to H_e, and the vertical component to V_e. Then the specimen will acquire a uniform induced magnetization with horizontal and vertical components of intensity

$$\left.\begin{aligned}\gamma_x &= k_x H_e,\\ \gamma_z &= k_z V_e,\end{aligned}\right\} \tag{44}$$

where k_x, k_z are the volume susceptibilities in the two directions.

For cylindrical specimens the field at the magnetometer can be calculated from the expressions given in the previous sections. Then, from the magnitude of the observed field, the values of the susceptibilities can be calculated.

The actual measurements are made in all cases in the normal way, that is, by raising and lowering the specimen; this is done with different values of the external field, for instance, $\pm 0{\cdot}03$ G. In this way it is only the magnetism of the specimen and the moving piston which is measured. It is impossible to make such measurements by keeping the specimen at rest and changing the field, since this changes also the magnetization of all other material in the neighbourhood of the magnetometer, and so alters the zero position of the spot; in addition, there is the direct effect of the field on the magnetometer due to lack of perfect astaticism. When the external field is applied with the specimen in the lower position, the deflexion of the magnetometer due to the finite astaticism is compensated where necessary by applying a suitable magnetic gradient derived from a special coil above the magnetometer on the roof of the hut, so as to bring the spot back near its normal zero.

In principle, a change in the vertical field should not deflect the magnetometer. However, owing to slight deviations of the V-coils from geometrical perfection some small dependence of scale reading on the vertical field V_e is usually found. This is generally, however, much less than the dependence on horizontal field; switching off the current in the vertical coil, so as to impose the normal earth's field (0·44 G in Manchester), will often leave the spot still on the scale. Because of this it is often convenient to measure the susceptibility of a specimen by traversing it off centre and measuring the induced vertical dipole.

When the specimen is of cylindrical form, the expressions (39) to (43) for the field at the magnetometer can be used directly with, of course, γ_x and γ_z given by (44).

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If, as is often the case, the distance z of the specimen from the lower magnet is comparable with the distance between the two magnets of the magnetometer, it is necessary when absolute values of the susceptibilities are required, to evaluate the field at both magnets and take the difference.

4·4. *Magnetization induced in specimen by field of magnetometer*

The field of the magnetometer will induce magnetism in the specimen and this will produce a field at the magnetometer and so, in general, a deflexion. Even when the specimen is of a simple geometrical shape an exact calculation of this effect is complicated. However, the limiting case when the specimen is a symmetrical body, for instance, a sphere of radius a small compared with the distance z of its centre from the lower magnet P_1, can be worked out exactly. For simplicity the effect of P_2 will be neglected (i.e. it is assumed that $z \ll L$). Further, the length of the magnet P_1 will be taken as very small.

The field of the magnet P_1 at the sphere will be sensibly uniform and of magnitude P_1/z^3, whence the induced dipole p will be given by

$$p = P_1 kV/z^3, \tag{45}$$

where V is the volume of the sphere and k is its volume susceptibility.

If the centre of the body is on the axis of the magnetometer, considerations of symmetry show that the field will be in the plane zOy (figure 13) of the magnetometer and so will produce no deflexion. However, when the cylinder is displaced off the axis to a position (xy), there will in general be a component B_x of this field at right angles to the plane of the magnetometer, and so a deflexion will result. The deflexion can be considered a consequence of the attraction or repulsion (according as k is positive or negative), exerted by the induced magnetism of the sphere on that pole of P_1 which is nearest to the centre of the sphere.

If the dipole P_1 of the lower magnet is parallel to Oy as in figure 12, and if the body of the sphere is at the point A at (xy), then the field B_x on the magnetometer due to the induced magnetization of the body can be calculated as follows. We assume x and y small compared with z. Denoting the distance AP by R, where $R^2 = x^2+y^2+z^2$, and the angle between R and Oy by θ, the potential of P_1 at A will be $U = P_1 \cos\theta/R^2$. The components of the field at A parallel to Ox, Oy and Oz will be $H_x = -\partial u/\partial x$, $H_y = -\partial u/\partial y$ and $H_z = -\partial u/\partial z$. These three components of the field will induce in the body three components of polarization which can be shown to be given by

$$p_x = 3P_1 kVxy/R^5,\ p_y = -P_1 kV/R^3,\ p_z = -3P_1 kVy^3/R^5. \tag{46}$$

The magnetic fields at P along Ox produced by the three components of polarization can then be shown to be $+3P_1 kVxy/z^8$, $+3P_1 kVxy/z^8$ and $-9P_1 kVxy/z^8$ respectively, and so the total resultant field B_x acting on the magnetometer

$$B_x = -\frac{3P_1 kVxy}{z^8}. \tag{47}$$

If the specimen is traversed along any straight line through the origin, specified by $y = \alpha x$, we have B_x proportional to $xy = \alpha x^2 \propto r^2$, where $r^2 = x^2+y^2$. Hence the deflexion of the magnetization will be a quadratic function of the displacement along any line through the origin.

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When a large specimen is placed close to the magnetometer calculations become difficult. However, simple physical considerations show that one may still expect a relation of the form

$$B_x \propto P_1 k\, xy, \tag{47a}$$

though the variation with z will be complicated. We therefore again expect a quadratic relation between the deflexion and the displacement along any line through the origin, except along the axis, when there will be no deflexion.

Since the deflecting field is proportional to P the effect becomes important with strong magnetometer magnets, and it has already been shown that strong magnets are required to measure very small fields; in fact, this induced field may become a sufficiently disturbing factor to prevent the use of very strong magnetometer magnets, and so set a limit to the sensitivity achievable.

In general, however, B_x can be made negligibly small by making r small, that is, by accurate centring of the specimen on the axis of the magnetometer. Such accurate centring is clearly a necessity in many experiments, even apart from the effect of the induced field. It will be remembered that the ram carrying the specimen is attached to a two-dimensional traverse mechanism, so as to allow the necessary movement for the centring process. The problem of how in practice to centre accurately the ram on the magnetometer axis appeared initially somewhat formidable. However, an elegant solution arose in the following way.

4·5. *Ballistic method of centring piston*

An attempt was made to use equation (47 *a*) as the basis of a centring procedure. One had only to find that co-ordinate position in which B_x is zero and where it remains zero for small displacements along the north-east to south-west and the north-west to south-east directions to achieve the desired result.

However, when specimens have a susceptibility of the order of 10^{-6}, as is the case for metals and most other materials, the deflexions obtained are too small to allow accurate centring. Strongly paramagnetic salts with $k \sim 10^{-4}$ would be adequate, but are inconvenient to use.

However, it was noted during an experiment with a copper cylinder that, whenever the cylinder was raised or lowered a large ballistic swing of the magnetometer was produced, due clearly to the currents induced in the cylinder by the field of the magnetometer. These deflexions were found to vary with x and y according to (47 *a*), that is, the deflexion was zero when the axis of the cylinder was in the plane zOx or zOy (figure 13), and varied quadratically with the horizontal displacement r along any line not on these axes. In fact, during its vertical motion the copper cylinder behaves like a highly diamagnetic body, owing to its high conductivity. Provided the vertical motion had a duration short compared with the period of swing of the magnetometer the swing is truly ballistic, i.e. the swing is independent of the rate of movement.

An approximate calculation can easily be made of the ballistic swing of a magnetometer when a small conducting body of dimensions a and conductivity ρ is raised quickly from a large distance to a distance z below the lower magnet of the magnetometer. It can be

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shown that the resulting ballistic swing is equal to that steady deflexion which would be produced if the body had a diamagnetic susceptibility k' given by

$$k' \sim a^2/\rho T, \tag{48}$$

where T is the period of the magnetometer. This expression can be obtained directly by a dimensional argument.

For copper $\rho = 1\cdot8 \times 10^{-6}$ ohm cm $= 1\cdot800$ e.m.u. If $a = 5$ cm and $T = 30$ s we have $k' \sim 10^{-4}$.

The observed ballistic swing is found experimentally to be of a magnitude corresponding to an effective diamagnetic susceptibility of the above magnitude. This procedure provides a quick and accurate method of centring a conducting cylinder under the magnetometer and so centring the piston carrying the cylinder. The swing is observed for various values of the co-ordinates of the ram, e.g. at the four points defined by the x and y co-ordinate values $(+1+1)$, $(+1-1)$, $(-1+1)$, $(-1-1)$. A simple plotting method then serves to define the point at which the centre of the cylinder is on the magnetometer axis. It is easy to achieve an accuracy of 0·2 mm by this method. Figure 14 shows an example of the variation of ballistic swing with a copper cylinder traversed along the north-east to south-west line.

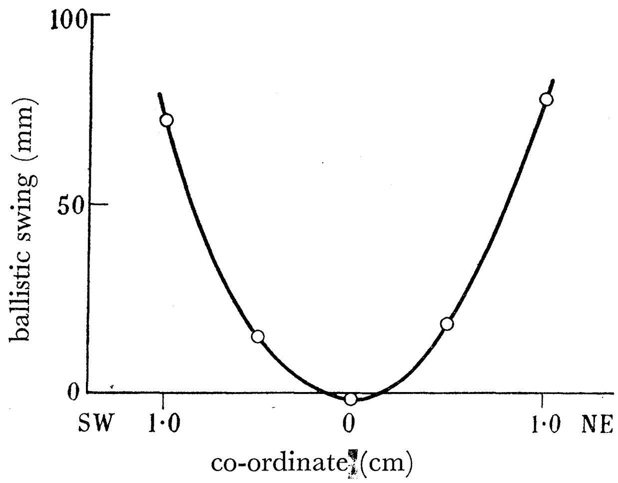

FIGURE 14. Ballistic swing of magnetometer when 10 cm copper cylinder is raised up below it, plotted against co-ordinate displacement in north-east to south-west direction.

The relative electrical conductivity of two cylinders is given by the ratio of their ballistic swings. For example, in one experiment the observed ratio of the swing due to a copper cylinder to that due to a lead cylinder was 11·5, while the text-book value of the ratio of conductivity is 11·8.

When the piston has been centred by this method, the copper cylinder is removed and the specimen under investigation, for instance, a geological specimen, is put in its place.

5. EXPERIMENTAL RESULTS

5·1. *Method of making measurements*

Suppose the magnetic field due to a weakly magnetized specimen mounted on the ram is to be measured. A reading d_1 of the spot is taken with the specimen in its lowest position; the specimen is raised to its upper position close under the magnetometer, and then after

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the time T which the spot takes to come to rest another reading d_2 is taken; the specimen is then lowered and after another time T a third reading d_3 is taken. The *deflexion* Δ due to the specimen is then taken as

$$\Delta = d_2 - \tfrac{1}{2}(d_1 + d_3). \tag{49}$$

This procedure is repeated say n times, thus giving n values of Δ. This method of calculating Δ eliminates the effect of a drift which is linear with time.

If d_1, d_2 and d_3 all have the same standard error δd, the standard error $\delta\Delta$ of Δ is shown in appendix 2 to be given by

$$\delta\Delta = \surd(\tfrac{3}{2})\,\delta d = 1{\cdot}23\delta d.$$

The *drift* D of the spot in the time T is defined as

$$D = \tfrac{1}{2}(d_1 - d_3), \tag{50}$$

since the time between readings d_1 and d_3 is $2T$.

In practice it is convenient to test the performance of the magnetometer in the first instance by omitting altogether the signal, that is, in the case above, the movement of the specimen. Then when n is large, the *algebraic* mean value $\overline{\Delta}$ of all the deflexions will be nearly zero, and the *arithmetic* mean A of all the n deflexions in the absence of a signal will be used to measure the accuracy of a single reading. It is found in practice, as would be expected, that provided the drift rate does not change too fast the calculated deflexions are independent of the drift; in fact, the possibility of obtaining a high sensitivity with the instrument depends essentially on the fact that significant values of Δ can be obtained in the presence of a drift D which may be many times greater than the average error A of a single reading.

It is found that the mean error A of a deflexion tends to be large (a) when magnetic disturbances, as measured by the fluxgate recorder, are large, (b) when there is a strong wind, (c) when it is raining.

5·2. *Diurnal and other variations of the random errors*

To get some systematic data on the variation of A with the conditions, eighteen deflexions without any signal were measured every 2 h for the 6 days 24 to 29 January 1951, by the second procedure described in appendix 2, so that each calculated deflexion was independent. This series of measurements was made by J. M. Pickering and Miss M. Almond. Of these deflexions nine were made with the fluxgate compensation in use and nine without. The Caslox-Parastatic magnetometer was used, with a gradient sensitivity g' of 3×10^{-8} G/mm. Though the free period T of the magnetometer was 45 s, for convenience readings were taken at intervals of 30 s, that is, $0{\cdot}66T$ (see § 2·1). The H and E components of the astaticism were about 2500 and 4000 respectively.

The open circles in figure 15 give the mean values of A in microns for each bi-hourly epoch. Each point then is the mean of $6\times18 = 108$ deflexions.

No systematic difference was found between the errors with and without the fluxgate compensation. This result can be explained, as will be shown below, by the relatively quiet magnetic conditions during most of the run.

The average value of A around noon is about $140\,\mu$ and around midnight about $70\,\mu$. These figures can be compared with the average error of a single deflexion due to the Brownian movement of the magnetometer, as calculated from (30) of § 3·4 to be $15\,\mu$. The observed error at night is thus 4·6 times the thermal error.

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Especially quiet periods occurred between 0000 and 0400 h on 28 January, between 2200 h on 28 January and 0600 h on 29 January, and between 1800 and 2400 h on 29 January. The average error of 216 readings was 45 μ, or three times the thermal error. The weather on these two days was calm and clear. Magnetically the nights were fairly quiet.

The black circles in figure 15 give the algebraic mean of each group of 108 readings. The relation between the errors of these mean values and the error of a single reading is discussed in the next section.

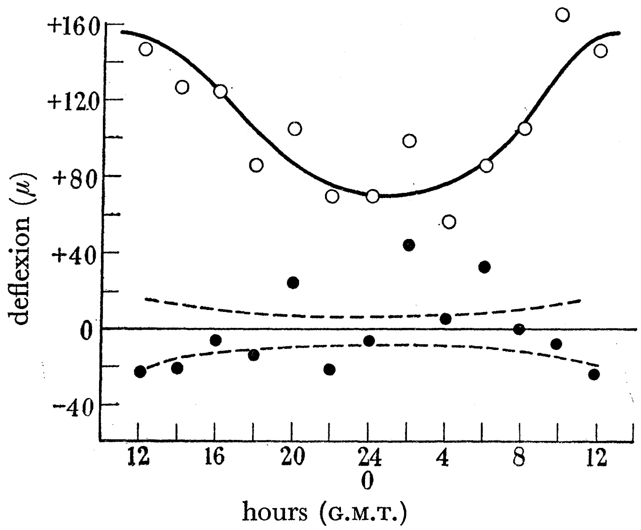

FIGURE 15. Diurnal variation of error of Caslox magnetometer. ○ and full curve, mean error of single reading; ● and dotted curve, algebraic mean of 108 readings.

It is probable that during the night the actual errors of the magnetometer were rather less than the above-mentioned figures suggest, since the error of *reading* the spot position due to the width of the diffraction image was certainly appreciable. An eyepiece scale with one division representing 40 μ was used and readings were made to integral divisions—hence the mean error of a single *reading* d due to this cause was about 20 μ, and so the mean error of a single *deflexion* Δ should have been about $20 \times 1{\cdot}23 = 25\ \mu$, that is, more than half the observed minimum of 45 μ.

In order to narrow the fringes the central vertical strip across the mirror was covered up; even so the width of the central fringes was about 400 μ. The thermal deflexion of 15 μ corresponds therefore to 4 % of the fringe width and the 20 μ error of a reading to 5 %. These figures emphasize the limitation set by the size of the mirror (1 cm diameter in this case), and show the need for some improved method of reading (see § 2·7). The 20 μ error of a reading corresponds to an angular deflexion of the mirror of 2×10^{-6} radian.

From the measured gradient sensitivity it is seen that the 45 μ error of a single reading on especially quiet nights corresponded to a field of $1{\cdot}4 \times 10^{-9}$ G.

Comparable, and sometimes much better, results were obtained with the other magnetometers. For instance, the set of readings made with the Alcomax magnetometer, in connexion with the static-body experiment, serve to give a typical value for A under favourable operating conditions. For instance, the sixty-four readings listed in the last two big

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columns of table 10 yield a mean deviation A from the mean of 57μ, which is close to the calculated value of 45μ for the thermal motion of the system (see table 5).

These results show that our magnetometers, under favourable conditions, are capable of approaching the designed performances as given in earlier sections.

During the whole 6-day run with the Caslox magnetometer the fluxgate record showed moderately quiet conditions. There appeared to be little or no correlation of the error with the relatively minor magnetic disturbances which were present; nor did these disturbances show any marked diurnal variation. Thus the observed variation of the errors appears not to be a result of those magnetic disturbances which are measured by the fluxgates.

The two results reported above, (*a*) the equality of the mean errors of the readings with and without the use of the fluxgate to compensate the field changes, and (*b*) the lack of correlation of the errors with the disturbances of the fluxgate trace which records changes in the earth's field, were somewhat unexpected.

A partial explanation seems to be that indicated in § 3·7. On normally quiet days the second time differential d^2H/dt^2 of the *uniform* component of the earth's horizontal field is sufficiently small for no appreciable errors to be introduced, provided the astaticism of the magnetometer is more than 2000 or so. Thus no reduction in errors is achieved by field compensation on a quiet day.

The fact that the observed errors are generally considerably larger than the calculated Brownian error, and that the errors are very variable, must therefore be sought in other causes than changes in the earth's horizontal field. These observed errors may be due to (*a*) changes in the second time derivative d^3H/dt^2dz of the vertical gradient of the earth's horizontal field, (*b*) gross thermal disturbances affecting the magnetometer, (*c*) mechanical disturbances. The present data are insufficient to indicate the relative importance of these three factors.

In figure 16 is shown the drift of the Caslox magnetometer spot on four separate occasions. The position of the spot was read in divisions of the microscope scale (12·5 div. to 1 mm) every 30 s. The particular records reproduced were chosen to show the nature of the drift under different conditions, ranging from very quiet to rather disturbed. Against each record is given the corresponding value of the arithmetic mean error A. For the quietest period $A = 0{\cdot}7$ div. $= 55\mu$, and for the most disturbed $A = 6{\cdot}1$ div. $= 490\mu$.

5·3. *Systematic errors and the error of the arithmetic mean*

An inspection of a long series of deflexions measured consecutively reveals that errors of successive deflexions are not completely independent, but sometimes show a systematic correlation over several successive deflexions. As a consequence of this it is not always valid to use the usual $n^{-\frac{1}{2}}$ factor to obtain the error of the arithmetic mean from the error of a single reading.

Direct evidence for this breakdown of the $n^{-\frac{1}{2}}$ rule is seen in figure 15. Each open point gives the *arithmetic* mean of 108 readings, taken in groups of eighteen at the same time of day on each of 6 days. The black circles give the corresponding *algebraic* mean. If all the 108 readings had been independent, one would have expected the algebraic mean to be on the average about $1/108^{\frac{1}{2}} = 1/10{\cdot}4$ of the arithmetic mean; the dotted curves (positive

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and negative) in the figure give these values. It is clear that in some cases the observed errors of the mean are considerably greater than expected.

It is peculiar that the discrepancy is greater at night than during the day; this appears to indicate that the small night errors are relatively less independent than the larger day errors. If this is a general result, the gain by working at night rather than by day, when many readings are taken and averaged, is less than would appear from a comparison of the errors of a single reading.

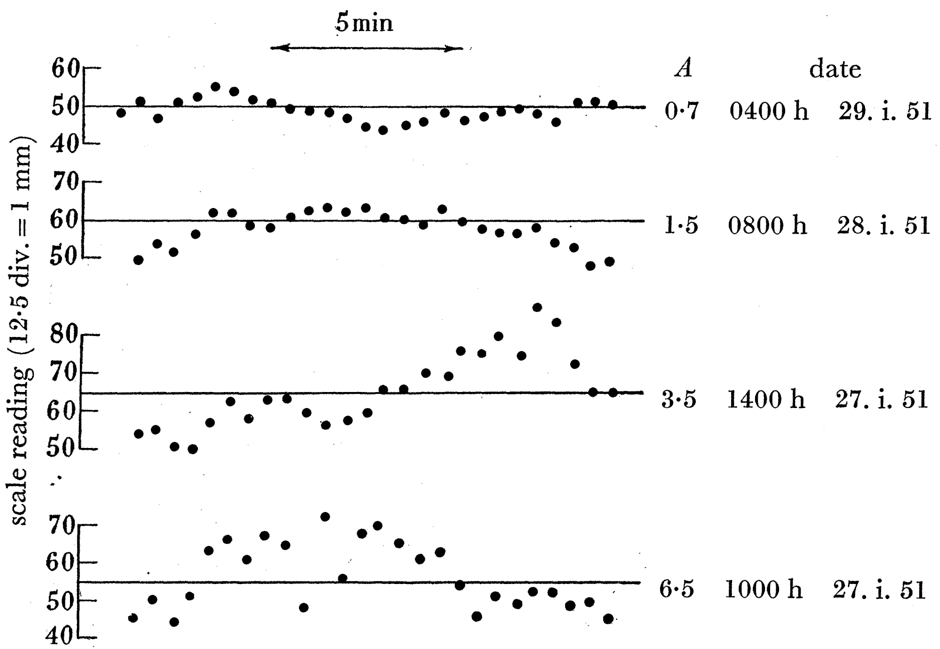

FIGURE 16. Drift of magnetometer spot in various magnetic conditions varying from very quiet (top plot) to rather disturbed (bottom plot). The value of A given against each plot is the arithmetic mean error calculated according to §5·1.

Taking all the 1296 readings together, we find a mean arithmetic error of 106μ and a mean algebraic error of 2μ; this very small value is certainly accidental, as the expected error assuming all readings to be independent is $106/(1296)^{\frac{1}{2}} = 3\mu$. This latter value corresponds to a field of $0{\cdot}9\times10^{-10}$ G.

The origin of such systematic errors extending over several readings, and therefore lasting up to perhaps as much as a quarter of an hour, is not fully understood. Some part, but probably not all, of such systematic errors may be due to the presence of a non-linear drift, the effect of which is not eliminated by the use of equation (49), but which could in principle be eliminated by the method described in appendix 2. Another possible source may lie in a small hypothetical semi-periodic swing of the spot with a period close to that of the time between successive readings.

Whatever the cause, the effect of these systematic errors can be eliminated without much waste of time by suitably spacing in time the readings corresponding to a given set of conditions. If, for instance, an accurate determination is required of the deflexion as a function of the azimuth of a specimen, a small number, say four, readings are taken on each azimuth in turn, and the whole set of readings are repeated as often as is necessary. Though the error of each individual reading of a set of four readings on some azimuth may

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not be independent, the mean of a set can be considered as independent of the mean of a subsequent set made after an interval of, say, a quarter of an hour, during which other azimuths have been investigated. Then the $n^{-\frac{1}{2}}$ factor can be applied to the mean error of the means of all the n sets of four readings to give the mean error of the grand mean.

Bearing this proviso in mind, the many results obtained with the magnetometer, for instance, those discussed above and those to be described in § 6, prove that by averaging a large number of deflexions it is possible to 'read through noise', that is, to obtain a final result of markedly greater accuracy than that of a single reading.

5·4. *Accuracy of measurement of magnetic properties of specimens*

According to the methods outlined in §§ 4·1 and 4·2, the magnetic properties of a cylindrical specimen are determined by measuring the magnetic field near its axis when the cylinder is set at a number of different azimuths. Neglecting all but the first harmonic we can write

$$H = H_c + H_p \cos(\chi + \chi_p), \tag{51}$$

where H_c is a constant term due to a number of possible causes, one of which is the effect of a vertical dipole and another the sought-for effect discussed in the next section, and H_p and χ_p are the amplitude and phase of the field of the horizontal dipole.

Suppose that a set of four determinations $\Delta_1 \dots \Delta_4$ of the deflexion of the magnetometer are made at equal intervals of azimuth χ, say 0, 90, 180, and 270°, giving with the sensitivity of the instrument four values of the field $H_1 \dots H_4$.

It can be shown by the usual methods (see, for instance, Brunt 1917, chapter 9) that the least-square solutions for the three unknowns are

$$H_c = \tfrac{1}{4}[H_1 + H_2 + H_3 + H_4], \tag{52}$$

$$H_p = [(H_1 - H_3)^2 + (H_2 - H_4)^2]^{\frac{1}{2}}, \tag{53}$$

$$\tan \chi_p = (H_2 - H_4)/(H_1 - H_3). \tag{54}$$

If now each of the four measurements of H are supposed subject to the same standard error σ, then it follows from the usual expressions for the random error of a function of several variables that the standard errors σ_1, σ_2 and σ_3 of the quantities H_c, H_p and χ_p are

$$\sigma_1 = \tfrac{1}{2}\sigma, \tag{55}$$

$$\sigma_2 = \sigma/\sqrt{2}, \tag{56}$$

$$\sigma_3 = \sigma/H_p. \tag{57}$$

It will be noticed that the minimum number of deflexions at equally spaced azimuths to give a determination of H_c, H_p and χ_p is four. If more readings are made, say $4m$ in number, it is irrelevant to the accuracy of the three constants whether one reading is made at $4m$ equally spaced azimuths or m readings at four equally spaced azimuths, provided, of course, that the function is truly harmonic.

5·5. *Duration of a determination of dipole strength*

Since a single determination of a deflexion involves three readings of the magnetometer spot—one with the specimen lowered, one with it raised and one with it lowered again—

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the duration of a single determination of a deflexion will be $2T'$, where T' is the sum of the time of swing of the spot and the time to raise or lower the specimen—this latter being less than 5 s with my apparatus.

When a number of successive deflexions are measured by the first procedure of appendix 2, the duration of a single determination of a series is $2T'$, and so the duration of, say, eight determinations is $16T'$. For $T' = 30$ s we have a total duration of 8 min. The time taken to change the azimuth of the specimen between readings—a few seconds—has been neglected. When the second procedure of appendix 2 is used the time taken to obtain the eight independent deflexions will be 12 min.

5·6. *Test measurements of field of small magnets and of geological specimens*

As a test of the apparatus a small test magnet, consisting of a length of 0·5 mm of Vicalloy wire 75 μ in diameter and with a dipole moment of 2×10^{-4} c.g.s., was placed on the piston and centred on the axis of the magnetometer, and the field acting on the magnetometer

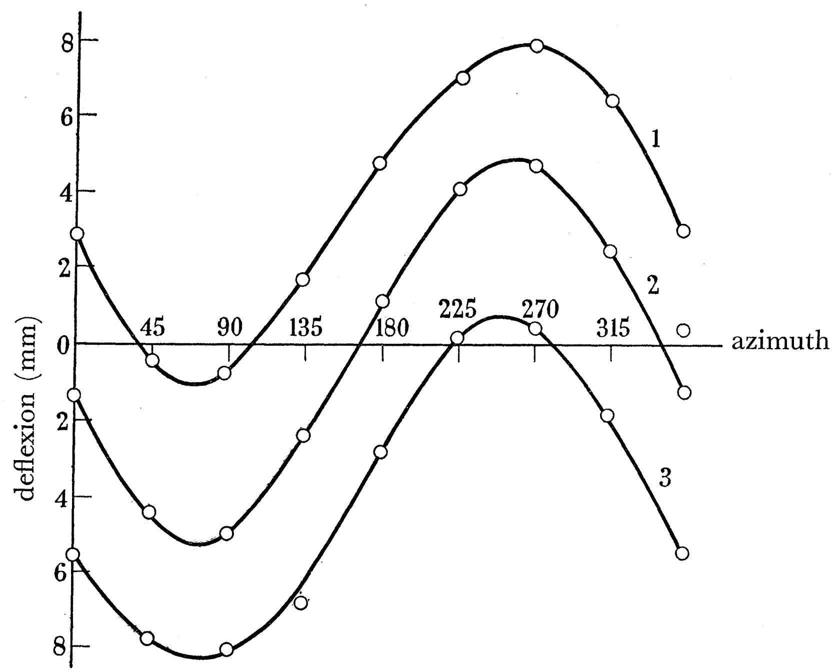

FIGURE 17. Magnetometer deflexion plotted against the azimuth of a thin cylindrical specimen of pre-Cambrian sandstone. Diameter 3·5 cm, thickness 0·8 mm. Curve 1, centre of cylinder 0·5 cm north; curve 2, on axis; curve 3, centre of cylinder 0·5 cm south.

was measured for different azimuths. The curves obtained were simple harmonic as expected. The measurements were repeated at various distances below the magnetometer, so checking the expected variation with distance, and also with the test magnet at different inclinations to the vertical and displaced 1 cm north and south of the central position, so checking the expressions given in § 4·1.

The magnetometer has been used to measure the permanent magnetism of various sedimentary rocks. The curves of figure 17 show the deflexions obtained with a cylindrical specimen of Torridonian sandstone of radius 1·75 cm and 0·8 mm thick prepared by Mr E. Irving. The middle curve shows the variation with azimuth when the specimen is

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centred under the magnetometer at a distance of 2·56 cm below the lower magnet of the Alcomax magnetometer. The upper and lower curves were obtained with the specimen moved 0·5 cm north and south of the centre.

The displacement of the curves is due to the effect of the vertical component of the magnetization of the specimen. From the curves the amplitude for the central position is 4·98 mm, which with the gradient sensitivity of $1{\cdot}75 \times 10^{-8}$ G/mm gives a field amplitude of $8{\cdot}72 \times 10^{-8}$ G. Applying first the simple dipole theory (equation (33)), we find an equivalent horizontal dipole of $1{\cdot}46 \times 10^{-6}$ c.g.s. The change of deflexion due to the traverse from 0·5 cm south to 0·5 cm north is 7·71 mm. Using (36), putting $x = 1$ cm, we find $\phi = 53°$.

Using the exact theory we have $v = a/z = 0{\cdot}68$ and $u = 0{\cdot}08/5{\cdot}12 = 0{\cdot}0156$. For these values we find $F_x = 0{\cdot}550$ and $F_z = 0{\cdot}371$, so $F_x/F_z = 1{\cdot}49$ (see also table 6) whence, from (39), we find that the total horizontal dipole of the specimen is $2{\cdot}66 \times 10^{-6}$ c.g.s.; dividing by the volume the horizontal intensity per unit volume is found to be $3{\cdot}45 \times 10^{-6}$ G. Using (43) we find for the angle of dip ϕ the value 63°. These calculations are given merely as an example of the method. Actually the particular specimen was not uniformly magnetized, so that the results given are not accurate. A precise method of treating non-uniformly magnetised specimens can, however, be worked out.

5·7. *Measurement of the permanent magnetism of non-ferrous metal cylinders*

In view of the intention to carry out the static-body experiment to be described in § 6, 10 cm cylinders were made of gold, lead, copper, brass and aluminium, and 5 cm cylinders of lead and of an 80 % tin and 20 % lead alloy. The machining was done with non-magnetic tungsten carbide tools. The pure gold cylinder was made by Messrs Johnson Matthey, as were also the 5 cm cylinders of pure lead and of tin-lead alloy. The other cylinders were made in the laboratory workshop.

After careful cleaning to remove surface ferromagnetic impurities some residual magnetism was always found, but it varied greatly for different cylinders, e.g. from 10^{-9} to 10^{-7} c.g.s./g. Moreover, the magnetism in some cases seemed to change with time, possibly through a self-annealing process or a gradual demagnetization caused by stray alternating fields. In this connexion it is known that the ferromagnetic properties of iron impurities in copper wires depend strongly on heat treatment (references are given by Bates 1948, p. 389). Large increases in the permanent magnetism of a lead cylinder could be produced by applying a field of some tens of gauss. A systematic study of the hysteresis properties of the magnetization could easily have been made and might have proved of interest, but they were not made through lack of time.

Some typical results will now be given.

A set of measurements with the gold cylinder gave a simple harmonic variation of deflexion with azimuth. The amplitude was $2{\cdot}45 \times 10^{-8}$ G at 7·7 cm above the *centre* of the cylinder, and $0{\cdot}83 \times 10^{-8}$ G at 10·6 cm. Calculating the horizontal dipole moment from the relation $p = z^3 H_1$, we get for the two heights the values $1{\cdot}12 \times 10^{-5}$ and $1{\cdot}00 \times 10^{-5}$ c.g.s. Thus the variation with height is consistent with a resultant horizontal dipole at the centre of the cylinder of about $1{\cdot}06 \times 10^{-5}$ c.g.s. Since the mass of the cylinder is $1{\cdot}52 \times 10^4$ g, the permanent magnetization per unit mass is $1{\cdot}06 \times 10^{-5} \div 1{\cdot}52 \times 10^4$ or $0{\cdot}7 \times 10^{-9}$ c.g.s. g^{-1}.

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Since the diamagnetic mass susceptibility of gold is $0{\cdot}15\times10^{-6}$ c.g.s. g^{-1}, the magnitude of the permanent magnetization on this occasion equalled that induced magnetization which would be produced by an external field of $0{\cdot}7\times10^{-9}\div0{\cdot}15\times10^{-6}\doteqdot\frac{1}{200}$ G.

5·8. *Irregularly distributed magnetization*

A set of measurements was made with a lead cylinder with a relatively large ferromagnetic impurity. The variation of deflexion with azimuth was measured with the cylinder centred under the magnetometer and traversed 1 cm north and 1 cm south. The results are shown in figure 18. It will be seen that in the central position the function $\Delta = f(\chi)$ is nearly simple harmonic with a mean ordinate of nearly zero. On the other hand, in the off-centre positions the curves are very far from simple harmonic, and the mean ordinates differ considerably from zero. These results can be interpreted as showing that the permanent magnetization of the lead cylinder was irregularly distributed throughout its volume.

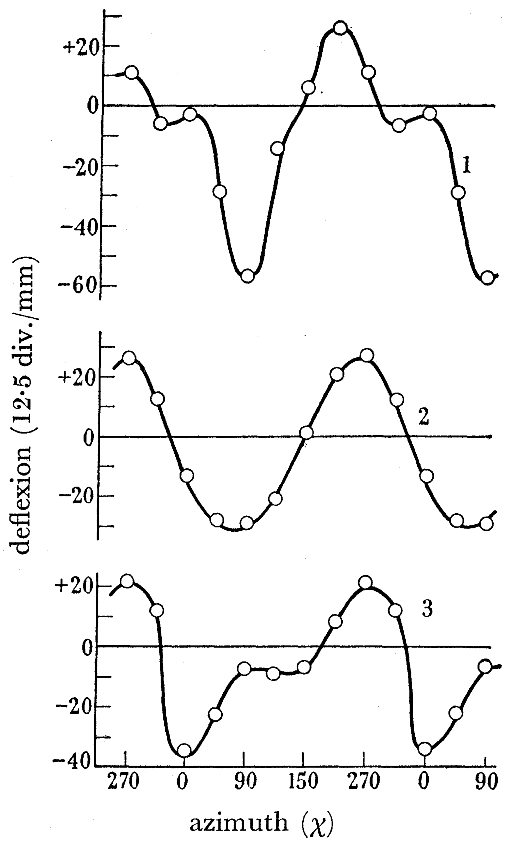

FIGURE 18. Magnetometer deflexion plotted against azimuth of 10 cm lead cylinder, showing effect of irregularly distributed permanent magnetism. Curve 1, 1·0 cm north; curve 2, on axis; curve 3, 1·0 cm south.

At any point on the axis of rotation of the specimen any distribution of irregular magnetization will give a resultant field, of which the horizontal component F will act on the magnetometer. As the specimen is rotated F will rotate with it but will remain constant in magnitude. Hence the deflexion Δ of the magnetometer will be a simple harmonic function of χ and its mean value will be zero, whatever the distribution of magnetization.

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When, however, the magnetometer magnet is not located on the axis of rotation of the specimen but at some distance off, the field will vary in a complicated manner as the specimen is rotated, since the various elements of the irregular distribution will be brought in turn nearer or farther away from the magnetometer. Thus the curves are not simple harmonic and the mean deflexion is not in general zero.

The fact that the mean deflexion is zero when the vertical axis of rotation of the specimen is coincident with the axis of the magnetometer, whatever the nature of the irregular distribution of the permanent magnetization, is of crucial importance for the static-body experiment described in § 6, for it allows the effect of any irregular magnetization to be eliminated.

5·9. *Susceptibility of metal cylinders*

The susceptibility of a number of cylinders was measured by the method described in § 4·3, that is, by applying a given horizontal field of the order of $\pm 0{\cdot}03$ G to the cylinder and measuring the induced horizontal dipole. In table 7 are given the measured susceptibilities relative to that of lead, corrected for the susceptibility of air ($+0{\cdot}03\times10^{-6}$ c.g.s.), together with the text-book values measured, of course, in much larger fields. All the cylinders were 10×10 cm. The values for mercury and manganese sulphate solutions were obtained by filling a hollow Perspex cylinder, 10×10 cm, with the liquid in question.

TABLE 7. MEASURED VALUES OF THE SUSCEPTIBILITY OF CYLINDERS OF DIFFERENT MATERIALS RELATIVE TO LEAD IN A MAGNETIC FIELD OF $\pm 0{\cdot}03$ G

The text-book values are given for comparison

	relative volume susceptibility	
substance	measured	text-book
lead	− 1·00	− 1·00
aluminium	+ 1·25	+ 1·26
mercury	− 2·16	− 1·88
gold	− 2·36	− 2·11
manganese sulphate, 833 g/l.	+47	+55

No great accuracy was attempted in these measurements, which were made only to test the reliability of the method and the purity of the substances. Any large impurity of ferromagnetic substances in a magnetically soft state would have been revealed by the measured values of susceptibility being found to be appreciably more positive than the text-book values for pure substances. No sign of such an effect is revealed by the figures.

The absolute values of the volume susceptibilities were calculated from the measured sensitivity of the magnetometer, using the theoretical expressions given in § 4·2 and table 6 for cylindrical specimens. No great precision was attempted. For gold the measured value was $-3{\cdot}11\times10^{-6}$. Correcting for the susceptibility of air we get the value $-3{\cdot}14\times10^{-6}$, while the text-book value is $-2{\cdot}90\times10^{-6}$.

Experiments were also made with two 5×5 cm cylinders, one of pure lead and one of 80 % tin and 20 % lead. According to the results of Honda & Soné (1913) the latter should have a susceptibility nearly zero. The measured value was less than 5 % of that of the lead cylinder.

I am not aware that the susceptibilities of materials have been measured before in such weak fields as 0·03 G.

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6. Static-mass experiment

6·1. *Plan of the experiment*

The experiment consisted of attempting to detect a small magnetic field of the order of 10^{-8} G in the neighbourhood of a stationary mass of heavy material, in practice cylinders 10 cm in diameter and 10 cm high of lead, copper and gold. The reasoning which led to this attempt was as follows.

Wilson (1923), following a suggestion by Schuster (1912), put forward the hypothesis that the magnetic field of the earth and the then believed magnetic field of the sun, could be explained if a neutral particle of mass M moving with velocity $\mathbf{v}$ produced a magnetic field H at a distance $\mathbf{r}$ given by

$$H = \beta \frac{G^{\frac{1}{2}}}{c} \frac{M\mathbf{v} \wedge \mathbf{r}}{r^3}, \tag{58}$$

in analogy with Biot-Savart's law for a moving charge. G and c are the gravitational constant and velocity of light, and β is a numerical constant with a value about 0·3 as deduced from the magnetic field of the earth.

Interest in Wilson's hypothesis was revived by Babcock's discovery of the magnetism of certain highly rotating stars, and by the lack of quantitative success of any theory of the origin of the fields. Recently, it has been found by Thiessen (1946, 1951), Klüber (1951) that the sun's measured field since 1945 has been less than one-twentieth of Hale's original value assumed by Wilson. If Hale's measurements are correct, then the sun must be a magnetic variable. Anyhow, the origin of the fields of the earth and stars have still to be explained.

Though relation (58) is clearly not true for a body moving with uniform velocity $\mathbf{v}$ past a stationary observer (see discussion by Blackett 1947, 1949*a*), it seemed conceivable that it might be valid for the case of a body and an observer sharing a common velocity $\mathbf{v}$ due to the rotation of the earth. In this case $\mathbf{v} = \mathbf{w} \wedge \mathbf{p}$, where $\mathbf{w}$ is the absolute angular rotation and $\mathbf{p}$ the vector distance from the axis of rotation to the body. Thus (58) could be written

$$H = \beta \frac{G^{\frac{1}{2}}}{c} \frac{M[\mathbf{w} \wedge \mathbf{p}]\,\mathbf{r}}{r^3}. \tag{58a}$$

Bullard pointed out that if this were true the earth's horizontal field below the surface would decrease downwards instead of increasing, as in all theories attributing the magnetism to the core. The quantitative predictions of these hypotheses were calculated by Runcorn (1948) and by Chapman (1948), and put to the test by Runcorn *et al.* (1950, 1951) by experiments in mines. For recent theoretical discussions see Papapatrou (1950) and Luschak (1951).

The results were decisively against this modified form of the Wilson-Schuster hypothesis and in favour of a 'core' theory. However, before Runcorn's experimental work was completed, I decided to put relation (58*a*) to a direct experimental test in the laboratory. After the experimental disproof by Runcorn and his colleagues of this relation by measurements in mines I continued the experiment, partly for the sake of an independent confirmation of their result, but partly as an exercise in the making of significant measurements of such small magnetic fields. The magnetometer itself was in fact designed specifically for this experiment.

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The main measurements were made with a gold cylinder of dimension 10×10 cm and mass 15·2 kg, which was mounted on the piston under the magnetometer, so that it could be raised or lowered a distance up to 13 cm from a given lower to a given upper position. The method of making the observation of the magnetometer spot has already been described.

If relation (58*a*) were true for any point mass, the field at a point on the axis of the cylinder of radius a and height $2b$ at a distance z from its centre would be given by integrating over the whole volume of the cylinder. I am indebted to Dr Papapetrou for this calculation which leads to the following result:

$$H = \beta \frac{2\pi v G^{\frac{1}{2}} \rho}{c} \{2b + [a^2 + (z-b)^2]^{\frac{1}{2}} - [a^2 + (z+b)^2]^{\frac{1}{2}}\}, \tag{59}$$

where ρ is the density.

When z is much greater than a and b this reduces to

$$H = \beta \frac{v G^{\frac{1}{2}}}{c} \frac{M}{z^2}, \tag{60}$$

where $M = 2\pi a^2 b \rho$, as it is obvious it must.

In the latitude of Manchester (50° N) v has the value of $2\cdot80 \times 10^4$ cm s^{-1}. Putting $a = b = 5$ cm, $\rho = 19\cdot3$ for gold, $G = 6\cdot8 \times 10^{-8}$ c.g.s. and $\beta = 0\cdot3$, we find the values of the field at different distances above the centre of the cylinder given in table 8.

TABLE 8. MAGNETIC FIELD AT A DISTANCE z CM ABOVE A VERTICAL GOLD CYLINDER 10×10 CM GIVEN BY EQUATION (59) FOR LATITUDE 53°

$z = 5$ cm corresponds to the top of the cylinder

z (cm)	5	7·5	10	12·5	15	20	25
H (10^{-10} G)	340	190	113	75	58	28	19

Since it has already been shown that the random error of a single reading of the magnetometer is about 10^{-9} G under favourable conditions, and that the random error of the arithmetic mean of a not unreasonable number of readings can be as low as 2×10^{-10} G, the essential experimental problem becomes that of eliminating all systematic errors. In the next subsection the main systematic errors will be discussed. Experimentally the attempt was made to reduce the systematic errors below about 2×10^{-10} G. Since the field given by (58*a*) would be in a north direction, the magnetometer magnets were set in the east-west direction.

In its upper position the top of the cylinder was usually some 2 to 3 cm below the lower magnet of the magnetometer; in its lowest position about 15 cm.

6·2. *Systematic errors*

6·2·1. *Ferromagnetic impurities*

Like all metal cylinders so far examined, the gold cylinder shows a very small but appreciable permanent magnetism which varies somewhat with time. The effect of this on the magnetometer is eliminated by the method outlined in § 5·8. The cylinder is centred accurately (to about 0·2 mm) on the axis of the magnetometer by the ballistic swing method

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(§ 4·5), and the field H due to the cylinder is measured for a number of equally spaced azimuths χ. The amplitude H_p of the resulting simple harmonic function

$$H = H_c + H_p \cos(\chi + \chi_p)$$

gives the effect of the equivalent horizontal dipole due to the usually unevenly distributed ferromagnetic impurities. Since the average value of the deflexion due to this permanent magnetization must be zero on the axis of the cylinder, the mean of the actual deflexions at equally spaced azimuths gives the deflexion H_c due to all other causes except that of the horizontal component of ferromagnetic impurities, and so eliminates the latter effect. It will be noted that in H_c appears the effect of any vertical magnetization when the cylinder is off axis.

To estimate a residual effect of the permanent magnetization due to inaccurate centring, the measurements of $H_c = f(\chi)$ are repeated with the cylinder traversed off-centre, first a distance $\pm 1·0$ cm in the north-south direction and secondly a distance $\pm 1·0$ cm in the east-west direction. From the values so obtained one can calculate the residual error due to a given estimated error δx, δy in centring. When the gold cylinder was traversed in this way, it was found that H_c changed by about 2×10^{-9} G for a 1 cm displacement. Since the centring was certainly better than 1 mm, the systematic error due to the contribution to H_c arising from bad centring was certainly less than 2×10^{-10} G.

6·2·2. *Magnetization induced by external field*

A direct test was made by applying a horizontal field of $100\,\gamma$ to the gold cylinder. The measured field at the lower magnet was 3×10^{-9} G. To keep this effect below 2×10^{-10} G requires therefore that the residual field at the centre of the coils should be less than about $6\,\gamma$.

The required degree of compensation of the earth's field should be obtainable using the fluxgate detector, since, as already mentioned, the maker's specification states that the fluxgate system should null the field to considerably less than $1\,\gamma$, and I have no reason to suppose that the performance was not attained. Since, however, it is difficult to check by direct experiment whether this was so, the following experimental procedure was used. The current in the H-compensating coils required to reduce the output of the fluxgate to zero was measured with a potentiometer. This was done in turn with all three fluxgate detectors placed in turn at the same position at the centre of the H coils. The values of the compensating currents so determined were found to agree to within 0·1 mA, which corresponds to a field of about $10\,\gamma$. Thus it can be concluded that the fluxgates do in fact compensate the field as least to about $10\,\gamma$. This is nearly sufficient, according to the above calculation, to keep the systematic error due to the induced magnetism of the cylinder down to about the acceptable limit.

Since the earth's field changes during even a quiet day by up to $50\,\gamma$, and much more on disturbed days, it is desirable to keep the field continuously compensated during an experiment by means of the fluxgate. When, for various reasons, it was not convenient to do this, reliance was placed on occasional determinations of the nulling values of the compensating currents.

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6·2·3. *Magnetization induced by field of magnetometer*

As shown in § 4·4 this effect is hard to calculate accurately for a body close to the magnetometer, but it can be expected to be of the form (47 *a*). In principle the effect can be measured by traversing the cylinder along the north-east to south-west or north-west to south-east lines, but with the low susceptibility of the gold cylinder it was not found possible, without undue labour, to demonstrate the effect. A subsidiary experiment was therefore made, in which the gold cylinder was replaced by a Perspex container filled with a strong solution of manganese sulphate, with a measured volume susceptibility of about 60×10^{-6}, that is, about 20 times that of gold. Traversing this along the 45° direction, the expected quadratic variation of magnetometer deflexion with displacement from the centred position was found. At 1 cm off the axis in a 45° direction the observed value of B_x with the manganese solution was 4×10^{-7} G. The corresponding field for the gold cylinder would be therefore 2×10^{-8} G at 1 cm off axis, and so 2×10^{-10} G at 1 mm off axis. Since the centring was certainly better than this, no systematic error due to this cause was to be expected.

6·2·4. *Other possible systematic errors*

If the axis of the piston, and so the motion of the cylinder, was not exactly vertical, a systematic error could be produced. No study of this effect was made; reliance was placed on accurate levelling, with a precision clinometer, of the table fixed to the top of the piston rod and machined accurately at right angles to its axis.

Systematic errors would be produced by non-uniform temperature distribution in the cylinder, resulting in thermal currents. After handling the cylinder such effects could be observed, but were almost certainly negligible in the actual experiments, when in all cases the cylinder was given time to attain a uniform temperature.

When continuous fluxgate compensation of the field components at the centre of the coils was not employed, the possibility existed that small changes in the earth's field would induce currents in the cylinder and so produce a field at the magnetometer. In some measurements such error was probably present, but could be eliminated by averaging a large number of readings taken at different times of day. For the sign of the effect would depend on whether the field was increasing or decreasing, and averaged over a day the effect should be zero.

6·3. *Experimental results*

The first tests of the modified Schuster-Wilson hypothesis were made during the spring and summer of 1949, using a lead cylinder and the Vectolite magnetometer. These measurements showed that there was certainly no field appreciably larger than that given in table 8. When the cylinder top was quite near (1 to 2 cm) the lower magnet, deflexions were observed of the order of or somewhat larger than the figures in the table, but varied in sign and magnitude on different runs. These anomalous effects decreased very rapidly with increasing distance of the cylinder from the magnet, and were therefore attributed to some induced magnetic effect of a more complicated type than that considered in § 4·4, possibly resulting from a combination of both imperfect centring and imperfect levelling. Further, the lead cylinder had a rather large ferromagnetism, which made measurements difficult.

During 1950 and 1951 a long series of measurements was made with the gold cylinder and the Caslox-Parastatic magnetometer, paying increased attention to centring and levelling

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the cylinder. The irregular effects in the highest position of the cylinder were much reduced. To eliminate possible unthought-of systematic effects depending on the field of the magnetometer itself, the magnetometer was periodically rotated through 180°; the double-sided mirrors already described allowed the spot to be read.

If the results are expressed as the ratio W of the observed field to that expected from table 8, this series of measurements extending over many weeks gave a value for W of 6 % south with a roughly estimated standard error of 13 %.

In addition, a series of readings was made with the magnetometer rotated into the north-south plane, so as to detect a possible east-west field. The result found for W was 12 % ± 15 % west. These measurements suggested that if any field of the type envisaged by Wilson existed, its magnitude was not more than 15 % of that expected.

With the experience gained in this work a series of measurements was made in the spring of 1951, using the gold cylinder and the Alcomax IV magnetometer. Two runs were made, the first between 2000 and 2400 h on 23 April, and the second with the gold cylinder inverted was made between 1500 h on 25 April and 0400 h on 26 April. The conditions were normally calm both meteorologically and magnetically. All the measurements were made by J. M. Pickering. A summary of the results is given in tables 9 and 10.

Table 9. Experimental results with gold cylinder. Run 1

Details are given in § 6·3

(a)	z (cm)	7·6		9·4		11·5	
(b)	n	16		8		8	
(c)	χ	Δ	σ	Δ	σ	Δ	σ
(d)	0	+1·60	1·25	−1·13	0·63	+0·69	0·69
	90	+1·54	0·51	+1·94	0·31	+0·69	0·45
	180	+2·42	0·48	−1·76	0·38	+0·57	0·57
	270	+0·82	0·85	+0·38	1·00	+2·13	1·37
(e)	means	+1·60	0·77	−0·16	0·58	+1·02	0·77
(f)	Δ_c	+1·60 ± 0·39 N		−0·16 ± 0·29 S		+1·02 ± 0·39 N	
(g)	H_c (10^{-10} G)	15·8 ± 3·8 N		1·6 ± 2·8 S		10·0 ± 3·8 N	

Table 10. Experimental results with gold cylinder. Run 2

(a)	z' (cm)	7·6		9·4		11·5	
(b)	n	16		8		8	
(c)	χ	$\bar{\Delta}$	σ	$\bar{\Delta}$	σ	$\bar{\Delta}$	σ
(d)	0	−1·88	1·15	−0·51	0·38	−0·50	0·0
	90	−0·68	1·09	+0·19	1·19	+0·63	0·38
	180	+1·54	2·33	+0·57	0·19	+0·07	1·32
	270	+3·20	1·50	+1·06	1·44	−0·19	0·57
(e)	means	+0·55	1·52	+0·33	0·80	0·00	0·57
(f)	Δ_c	+0·55 ± 0·76 N		+0·33 ± 0·40 N		0·00 ± 0·29	
(g)	H_c (10^{-10} G)	5·4 ± 7·4 N		3·2 ± 3·9 N		0·0 ± 2·8	

Measurements were made of the deflexion of the magnetometer at four equally spaced azimuths χ at different distances z of the centre of the cylinder from the lower magnet. For $z = 7·6$ (that is, when the *top* of the cylinder was 2·6 cm below the magnet), the vertical travel of the cylinder was 13·5 cm; for greater values of z the travel was correspondingly less.

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In the tables, row (*a*) gives the value of z, row (*b*) the number n of deflexions measured at each azimuth, always in groups of four, the groups being widely spaced in time. On the left of each column of row (*d*) are given the mean deflexions $\bar{\Delta}$ in microscope divisions of all the n deflexions taken at each of the four azimuths. The mean of each group of four deflexions will be called a determination. On the right of the column is given the standard error σ of these determinations, calculated from the expression

$$\sigma = \left(\frac{\Sigma r^2}{m(m-1)}\right)^{\frac{1}{2}}, \tag{61}$$

where r is a residual and $m = \frac{1}{4}n$ is the number of independent determinations. For $m = 2$ ($n = 8$) we note that $\sigma = \bar{r}$.

In row (*e*) are given the mean values of the determinations of the deflexion on the four azimuths and their mean standard error. Referring to the argument of § 5·4 we see that the constant term H_c in (51) is given by (52) and so equals the average of the four determinations on the different azimuths, while (55) shows that the standard error of H_c is one-half that of a single determination. Taking the latter to be the average of the standard errors of the four determinations, we obtain the values of the constant term Δ_c (corresponding to H_c) together with their probable error; these are given in row (*f*). Since the gradient sensitivity of the magnetometer was 1 mm $= 1{\cdot}20 \times 10^{-8}$ G, and there were 12·3 microscope divisions to 1 mm, 1 microscope division corresponds to a field of $9{\cdot}8 \times 10^{-10}$ G.

Using this conversion factor we obtain finally the volume of H_c given in units of 10^{-10} G with their standard errors in row (*g*). Since the north pole of the lower magnet of the magnetometer pointed west, a northerly directed field produced a leftward deflexion of the spot. In the sign convention used a leftward deflexion was counted as positive. Hence the directions of the measured field which are given in row (*g*).

The final values of the measured field for both runs are given in table 11 and figure 19. Inspection of the figures shows that the difference between the measured field for the same value of z is always somewhat larger than the error of the mean of the two determinations as calculated from their individual standard errors. This indicates that some systematic errors were probably present affecting the two runs differently. In view of this, the standard error of the mean of the two runs as given in the fourth column has been calculated from (61); that is, since here $m = 2$, the standard error of the mean is taken as equal to half the difference between the results for the two runs.

In the fifth column is given the magnetic field to be expected on the modified Schuster-Wilson hypothesis as derived from table 8, taking into account the distance between the magnets of the magnetometer and the movement of the cylinder. In the last column are given the values of the ratio

$$W = H_{\text{obs.}}/H_{\text{theor.}},$$

together with their probable errors. The mean of the three determinations, each weighted inversely proportional to its standard error, is

$$\overline{W} = (4{\cdot}2 \pm 2{\cdot}6)\ \%\ \text{north.}$$

In addition to these measurements with the gold cylinder, a set of measurements with the 10 cm copper cylinder was made by the writer on the evenings of 21 and 22 April 1952. The total time of the run was 5 h.

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Five sets of four deflexions were measured on the four azimuths, giving a total of eighty deflexions, all at the same value of z of 7·9 cm. Evaluating the results as before, the measured field was

$$(2{\cdot}7 \pm 5{\cdot}8) \times 10^{-10}\,\mathrm{G}\text{ north},$$

which corresponds to a value of W of

$$(4{\cdot}5 \pm 9{\cdot}8)\ \%\text{ north}.$$

We conclude from these results that if any field of the Schuster-Wilson type exists, its value is not likely to be much more than 5 % of that given by the expression (58a).

TABLE 11. COMPARISON OF OBSERVED MAGNETIC FIELD WITH THE PREDICTIONS OF THE RUNCORN-CHAPMAN MODIFICATION OF THE SCHUSTER-WILSON HYPOTHESIS

Details are given in § 6·3

z	$H_{\text{obs.}}$ (10^{-10} G) run 1	run 2	mean	$H_{\text{theor.}}$ (10^{-10} G)	$W = H_{\text{obs.}}/H_{\text{theor.}}$ (%)
7·6	15·8 ± 3·8 N	5·4 ± 7·4 N	10·6 ± 5·2 N	145	7·3 ± 3·6 N
9·4	1·6 ± 2·8 S	3·2 ± 2·8 N	0·8 ± 2·0 N	90	0·9 ± 2·2 N
11·5	10·0 ± 3·8 N	0·0 ± 2·8	5·0 ± 5·0 N	53	9·6 ± 9·6 N

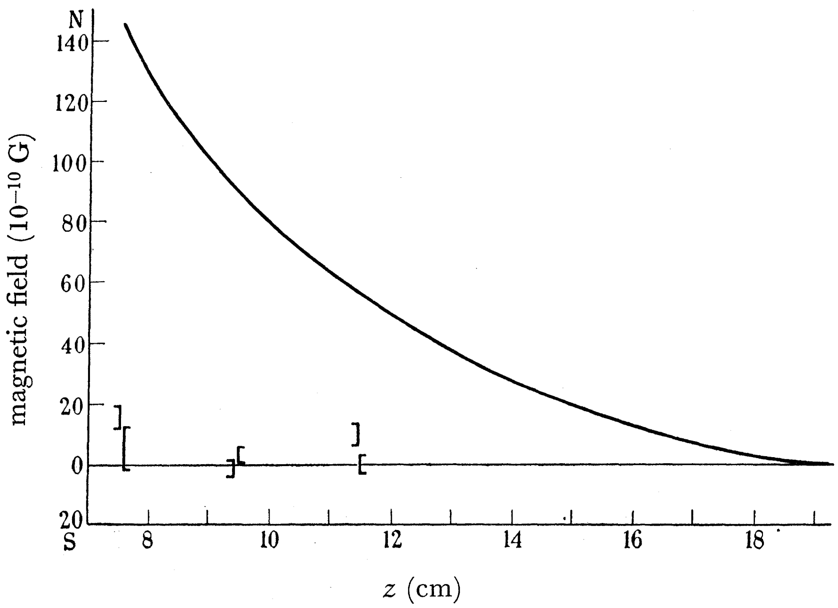

FIGURE 19. Comparison of measurements using the gold cylinder with field deduced from the Schuster-Wilson hypothesis.], run 1; [, run 2. The height of the vertical lines is twice the estimated standard error.

6·4. *Conclusion and possible improvements in the method*

With the experience gained in these measurements it would certainly have been possible to improve the accuracy of the final result. However, it was considered that the main object of the work had been attained. For it had been proved that no field of the Schuster-Wilson

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type, large enough to explain the magnetism of the earth, exists. So the experiments were discontinued.

Apart from mere repetition of readings, the following changes might lead to better results.

It was noticed that not only did the random error of a measurement tend to be larger when the gold or copper cylinder was close to the magnetometer, but indications of systematic errors maintained over a long set of readings were found. These increased random and systematic errors when the cylinder was very close to the magnetometer are not fully understood, but may perhaps be associated with the high electrical conductivity of the cylinder, leading to an electro-magnetic coupling between the pendulous mechanical oscillations of the magnetometer and its torsional oscillation. If the magnetometer has a slight pendulous swing, its field will induce currents in the cylinder; and the field of this current will react on the magnetometer. If the cylinder is not perfectly centred or levelled, it seems possible that a net couple might be produced leading to a systematic error depending sensitively on the centring and levelling.

As a result of these larger errors for small values of z it will be noticed from table 11 that the standard error of the final determination of W is lower for $z = 9{\cdot}4$ cm than for $z = 7{\cdot}6$ cm, in spite of the fact that at the smaller distance twice as many readings were taken and the expected field is 50 % larger. For large values of z, the error of W will be larger again owing to the reduction of the expected values.

Thus one can conclude that better results would have been obtained by making more measurements at values of z of about 9 to 10 cm.

If the greater random error for small values of z is, as suggested, due to the high conductivity of the gold and copper cylinders used, one would expect smaller errors using a material of lower conductivity, for instance, lead. Unfortunately, the two lead cylinders tried out had too much ferromagnetic impurity to be used satisfactorily.

Probably the largest source of systematic error lay in the field due to the magnetization induced by the imperfectly compensated external field (§ 6·2·2). This effect could be reduced by using a 'non-magnetic' alloy, for instance, 80 % tin and 20 % lead, for which a susceptibility of less than 5 % of lead can certainly be obtained (§ 5·9). On due consideration of these effects, I now consider that the choice of a gold cylinder for its high density was a mistake, and it would have been better to choose a non-magnetic tin-lead alloy. Another possibility which was given serious consideration was to use a heavy non-conducting material; the most suitable seems to be a heavy lead glass of density about 6·0.

Appendix 1. Magnetic properties of materials

In table 1 have already been given the stated properties of a selection of materials used for permanent magnets. Figure 20 gives the corresponding B-H curves. All the figures are those given by the manufacturers, except those for Vicalloy for which no figures were available, and so they were calculated from measurements made by the writer.

When a rod of magnetic material is magnetized to saturation and then removed from the field, its intensity of magnetization per unit volume falls to a residual value j which depends

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on the demagnetization coefficient D appropriate to the particular shape. This is defined by the relation

$$H_D = -4\pi D j, \tag{61}$$

where H_D is the field inside a body of uniform magnetization j. It can be shown (see Stoner 1946) that the value of j is that corresponding to a point on the B-H curve for which

$$\frac{B}{H} = \frac{1-D}{D}. \tag{62}$$

Denoting B_0, H_0 by these values, we have

$$j(\beta) = \frac{1}{4\pi}\frac{H_0}{D} = \frac{1}{4\pi}\frac{B_0}{1-D}. \tag{63}$$

The intensity of magnetization per unit mass $J(\beta)$ equals $j(\beta)/\rho$.

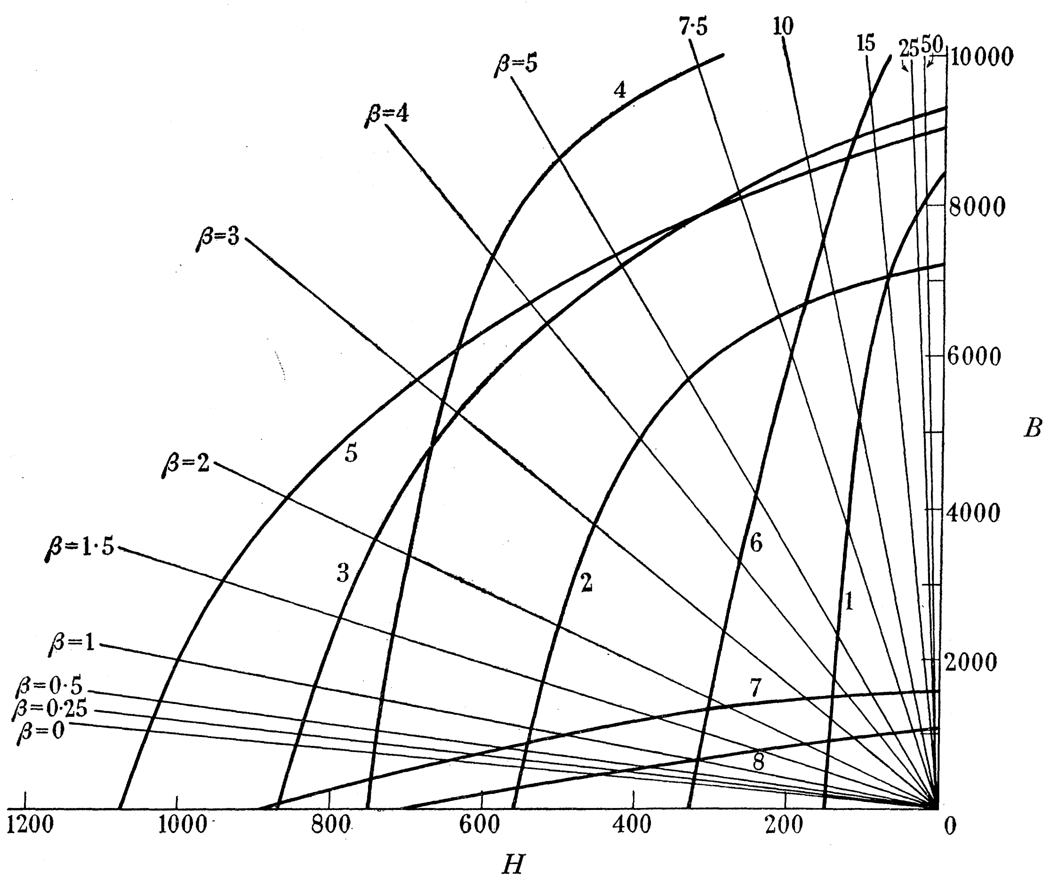

FIGURE 20. B-H curves for magnetic materials listed in table 1. The slope of the inclined lines is $(1-D)/D$ for various values of the fineness ratio β.

In table 12 are given values of D taken from the paper by Stoner (1946) for prolate ellipsoids with the ratio of the two axes equal to β. It would not be expected that any great error would result from using these values of D for ellipsoids to calculate the residual magnetization of rectangular magnets.

In figure 20 lines are drawn with the slope $(1-D)/D$, and from the points of intersection of these lines with the various B-H curves $J(\beta)$ can be calculated using (62). The resulting

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values of $J(\beta)$ are given in figure 3. No great accuracy is expected from these curves, owing partly to the approximate value of D used, but also to the considerable variation of magnetic properties of commercial materials. Individual specimens may and do deviate considerably from the maker's stated mean values.

The dipole moments of a number of rectangular specimens of the different materials and with various values of β were measured, and in general were found to agree with the curves of figure 3 to within $\pm 10\%$.

As no B-H curve for Vicalloy was available, the dipole moments of small lengths (0·5 to 16 mm) of 0·13 mm diameter Vicalloy wire were measured. From the resulting value $J(\beta)$ the B-H curve was deduced using (61), (62) and the values of D given in table 12.

TABLE 12. DEMAGNETIZATION COEFFICIENTS FOR PROLATE ELLIPSOIDS, TAKEN FROM STONER (1946), FOR DIFFERENT VALUES OF THE FINENESS RATIO β

β	50	25	15	10	7·5	5·0	4·0	3·0
D	0·0014	0·0049	0·0107	0·0203	0·0313	0·0558	0·0754	0·109
β	2·0	1·5	1·0	0·5	0·25	0·10	0	
D	0·174	0·233	0·333	0·413	0·463	0·490	0·500	

APPENDIX 2. NOTE ON METHOD OF MAKING OBSERVATIONS

1. *Alternative methods of making multiple readings*

Suppose N successive readings $d_1 \dots d_N$ of the spot are made with the object of determining as accurately as possible the deflexion of the magnetometer due to some signal which can be imposed at will.

Procedure 1. Assume N to be odd. The first reading d_1 is taken with no signal, the second d_2 with the signal, the third d_3 without, the fourth with and so on. From these N readings $n = \frac{1}{2}(N-1)$ deflexions. $\Delta_1 \dots \Delta_n$ are calculated from the expressions

$$\left.\begin{aligned} \Delta_1 &= d_2 - \tfrac{1}{2}(d_1+d_3), \\ \Delta_2 &= d_4 - \tfrac{1}{2}(d_3+d_5), \\ &\text{etc.} \end{aligned}\right\} \qquad (64)$$

These values of Δ, etc., are not independent, as each reading but the first and last is used twice. The mean value $\bar{\Delta}$ is given by

$$\bar{\Delta} = \frac{1}{n}[-\tfrac{1}{2}d_1 + d_2 - d_3 + d_4 - d_5 \dots - \tfrac{1}{2}d_N]. \qquad (65)$$

Suppose each of the N readings has a standard random error of magnitude δ. Then the standard error $\delta\Delta$ of Δ is found from the usual expressions (e.g. Brunt 1917) for the error of a linear function of several variables to be

$$\delta\Delta = \frac{2\sqrt{(N-\frac{3}{2})}}{N-1}\,\delta. \qquad (66)$$

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Procedure 2. Assume N to be a multiple of 3. The signal is now applied for the second, fifth, eighth, etc., readings, so that we obtain $n=\frac{1}{3}N$ *independent* deflexions:

$$\left.\begin{aligned}\Delta_1 &= d_2-\tfrac{1}{2}(d_1+d_3),\\ \Delta_2 &= d_5-\tfrac{1}{2}(d_4+d_6),\\ &\text{etc.}\end{aligned}\right\} \tag{67}$$

The mean value $\bar{\Delta}$ is given by

$$\bar{\Delta} = \frac{1}{n}[-\tfrac{1}{2}d_1+d_2-\tfrac{1}{2}d_3-\tfrac{1}{2}d_4+d_5-\tfrac{1}{2}d_6\ldots-\tfrac{1}{2}d_N]. \tag{68}$$

Since there are now $\frac{2}{3}N$ terms with coefficient $\frac{1}{2}$ and $\frac{1}{3}N$ with coefficient unity, we find that the error of the mean is now

$$\delta\Delta = \frac{3\delta}{\surd(2N)}. \tag{69}$$

When N has its minimum value of 3, equations (66) and (69) give, as they must, the same result $\delta\Delta = \surd(\frac{3}{2})\,\delta$. For any other given value of N, and so given total duration of the readings, the first procedure gives slightly the smaller value of $\delta\Delta$ and so a slightly greater accuracy. For N large the difference is 6 %.

We conclude that there is little in general to choose between the two procedures. The first gives more determinations of the deflexion, but these are not independent; the second gives fewer independent determinations.

2. *Numerical method of eliminating non-linear drift*

If the displacement d of the spot from some arbitrary zero is not a linear function of the time, the above procedures for calculating the value of Δ will not be accurate. If, however, the drift rate is observed without the signal, both before and after a reading with the signal, then it is clearly possible to use the two different drift rates to predict what the central reading would have been if no signal had been present, and so to eliminate the error due to the non-linear drift.

The simplest way of doing this is, of course, by plotting. However, the following numerical method may have some advantages.

Consider two successive readings d_1 and d_2 with no signal, one d_3 with signal, and two more d_4 and d_5 without. All five readings are to be made at equal intervals of time. We need to find an expression for the central reading d_3 in the absence of the signal, in terms d, d_1, d_2, d_4 and d_5. Assuming that the displacement of the spot is a quadratic function of the time, the required expression is found to be

$$d_3 = (d_3)_{24}+\tfrac{1}{3}[(d_3)_{24}-(d_3)_{15}], \tag{70}$$

where

$$(d_3)_{24} = \tfrac{1}{2}(d_2+d_4),$$

$$(d_3)_{15} = \tfrac{1}{2}(d_1+d_5).$$

The first term of (70) is the mean of the two neighbouring readings and is exact for a linear drift. The second term is the correction due to a non-linear drift, and equals one-third of the difference between the means of the inner and outer two readings.

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Appendix 3. Induction method of measuring weak magnetic fields

It is useful to compare the ultimate sensitivity of the astatic magnetometer for the detection of a weak steady field with that of a coil and amplifier for the detection of a weak alternating field. The treatment given here is a simplified version of that by Johnson (1938).

Consider an alternating field $H_0 \sin 2\pi ft$ at right angles to a circular coil of mean radius a wound with N turns of wire of specific resistance ρ and area of cross-section s. Then the resistance is

$$R = 2\pi a\rho N/s.$$

It is convenient to introduce the ratio $\alpha = Ns/\pi a^2$ which measures the ratio of the total area of the conductor to the area cross-section of the coil.

Then
$$R = 2\rho N^2/\alpha a \tag{71}$$

for a coil of square cross-section, with width equal to the radius of the coil, $\alpha = 1/\pi$. Allowing for a filling factor of say 60 %, α becomes about 0·2. Since the thermal noise in a resistance R has an r.m.s. value

$$E_n = 1{\cdot}27\times10^{-10}(R\Delta f)^{\frac{1}{2}}, \tag{72}$$

where Δf is the band-width of the amplifier (see Strong 1938, p. 436), the thermal noise is

$$E_n = 1{\cdot}79\times10^{-10}N\left(\frac{\Delta f\rho}{\alpha a}\right)^{\frac{1}{2}}. \tag{73}$$

The r.m.s. value of the e.m.f. in volts induced by the alternating field is

$$\left.\begin{aligned} E_s &= \pi a^2 N\, 2\pi f H_0\, 10^{-8}/\surd 2 \\ \text{or}\quad E_s &= 1{\cdot}39\times10^{-7}NfH_0a^2. \end{aligned}\right\} \tag{74}$$

If we define, as before, the minimum detectable field as that field which gives an induced e.m.f. equal to the thermal noise, we put $E_s = E_n$, and so we find

$$H_0 = 1{\cdot}29\times10^{-3}\frac{\rho^{\frac{1}{2}}(\Delta f)^{\frac{1}{2}}}{\alpha^{\frac{1}{2}}fa^{\frac{5}{2}}}. \tag{75}$$

The time of response T of the amplifier system, defined as the time taken for the output of the amplifier to reach within 1/e of its final value, is given by

$$T = 1/\Delta f,$$

whence
$$H_0 = 1{\cdot}29\times10^{-3}\frac{\rho^{\frac{1}{2}}}{\alpha^{\frac{1}{2}}fT^{\frac{1}{2}}a^{\frac{5}{2}}}. \tag{76}$$

Putting $\rho = 1{\cdot}7\times10^{-6}$ ohm cm for copper, and $\alpha = 0{\cdot}2$ say, and $T = 30$ s, we have

$$H_0 = \frac{3{\cdot}85\times10^{-6}}{fT^{\frac{1}{2}}a^{\frac{5}{2}}}. \tag{77}$$

These expressions for H_0 in terms of the constants of the apparatus are to be compared with equations (18) and (19) of § 2·4 for the magnetometer. Ultimately, the lowest attainable value of H_0 is limited in the former case by the resistivity of metal conductors at room temperature, and in the latter by the retentivity of existing magnetic materials.

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In these calculations it is assumed that the noise of the amplifier can be neglected compared with that of the coil itself. In practice it would generally be necessary to use two opposing coils of equal area-turns to reduce external disturbances. This will roughly double the total resistance and so increase H_0 by a factor $\sqrt{2}$.

Table 13 gives some values of H_0 for $T = 30$ s and for different values of the radius a of the coil and the frequency f, calculated by multiplying the values given by equation (77) by $\sqrt{2}$.

TABLE 13. VARIATION OF MINIMUM DETECTABLE ALTERNATING MAGNETIC FIELD WITH FREQUENCY AND RADIUS OF COIL, ASSUMING A PERIOD OF OBSERVATION OF 30 S

a (cm)	frequency (c/s) 10	100	1000
1	$1{\cdot}0 \times 10^{-7}$	$1{\cdot}0 \times 10^{-8}$	$1{\cdot}0 \times 10^{-9}$
3	$6{\cdot}4 \times 10^{-9}$	$6{\cdot}4 \times 10^{-10}$	$6{\cdot}4 \times 10^{-11}$
10	$3{\cdot}2 \times 10^{-10}$	$3{\cdot}2 \times 10^{-11}$	$3{\cdot}2 \times 10^{-12}$
30	$2{\cdot}0 \times 10^{-11}$	$2{\cdot}0 \times 10^{-12}$	$2{\cdot}0 \times 10^{-13}$
100	$1{\cdot}0 \times 10^{-12}$	$1{\cdot}0 \times 10^{-13}$	$1{\cdot}0 \times 10^{-14}$

From the figures in the table it can be seen that it is possible to detect much smaller alternating fields than steady fields, provided relatively large coils and high frequencies are used. For many applications, however, the radius of the coil is limited to a few centimetres in order to match the size of the specimen to be spun in it. The alternating field method has the advantage of not placing the specimen in an appreciable magnetic field, as may occur with the magnetometer.

We can make a rough estimate of the performance of Johnson's apparatus for measuring weak magnetizations as follows. If a dipole p is placed on and parallel to the axis of a coil of radius a and at a distance z from the coil, then it can be shown that the flux through the coil is $2\pi p a^2(a^2+z^2)^{-\frac{3}{2}}$. The uniform field H which gives the same flux is found by equating this expression to $\pi a^2 H$, whence

$$p = \tfrac{1}{2}H(a^2+z^2)^{\frac{3}{2}}. \tag{78}$$

For Johnson's instrument $a \doteqdot z \doteqdot 3{\cdot}5$ cm, $T = 4$ s and $f = 10$ c/s. Inserting these values in (77) we find $H_0 = 8{\cdot}6 \times 10^{-9}$ G and $p = 59H$. Putting $H = H_0$ we get $p = 5{\cdot}1 \times 10^{-7}$ c.g.s., which is close to the value of 5×10^{-7} c.g.s. quoted by Johnson. If the specimen is a 3 cm cube the lowest detectable intensity of magnetization will be given by dividing p by 27 cm^3, that is, $1{\cdot}9 \times 10^{-8}$ c.g.s.

To compare this performance with that of our magnetometer, suppose again the specimen in the form of a 3 cm cube is placed with its centre at a distance $z = 4$ cm below the bottom magnet of the Alcomax magnetometer, for which we will suppose, as in § 5·4, that $H_0 = 10^{-9}$ G. If eight independent deflexions in all are measured, then from (55) (putting $n = 2$) we find the minimum detectable dipole is equal to $4^3\,10^{-9}/4^{\frac{1}{2}} = 3{\cdot}2 \times 10^{-8}$ c.g.s. Dividing by the volume of 27 cm^3 we find the minimum detectable intensity of magnetization to be $1{\cdot}2 \times 10^{-9}$ c.g.s. The duration of such a determination will be about 12 min. The field of the magnetometer at the top surface of the 3 cm cubic specimen will be $P(z-1{\cdot}5)^3$ G, giving with $P = 15$ and $z = 4$ cm a field of $1{\cdot}0$ G.

It appears, therefore, that our magnetometer with $T = 30$ s has a theoretical minimum detectable dipole, or intensity of magnetization, which is about one-tenth of that of the

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Johnson instrument with $T = 4$ s. Thus with the same values of T their theoretical performances should be about the same.

The investigations described in this paper have been made possible by the skilled assistance of T. Ball, who constructed most of the mechanical parts of the apparatus, of A. H. Chapman, who made and installed all the electrical gear and helped me in many other ways, and of J. M. Pickering, who operated the instrument and made most of the measurements. I am indebted to J. Butterworth, of the Buildings Office of Manchester University, for the design of the magnetometer hut.

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My academic colleagues who have aided me by their helpful criticism of the first drafts of the paper are too numerous for all to receive individual mention. However, I owe a special debt to Dr A. Papapetrou for his constant interest in the main objective of the work described in this paper, that is, the carrying out of the static-body experiment; for deriving for me several essential expressions for the magnetic field of cylindrical bodies on various assumptions, and for his exceptionally careful reading of the draft paper. To Dr J. A. Clegg also I owe many thanks for the finding and correction of several errors.

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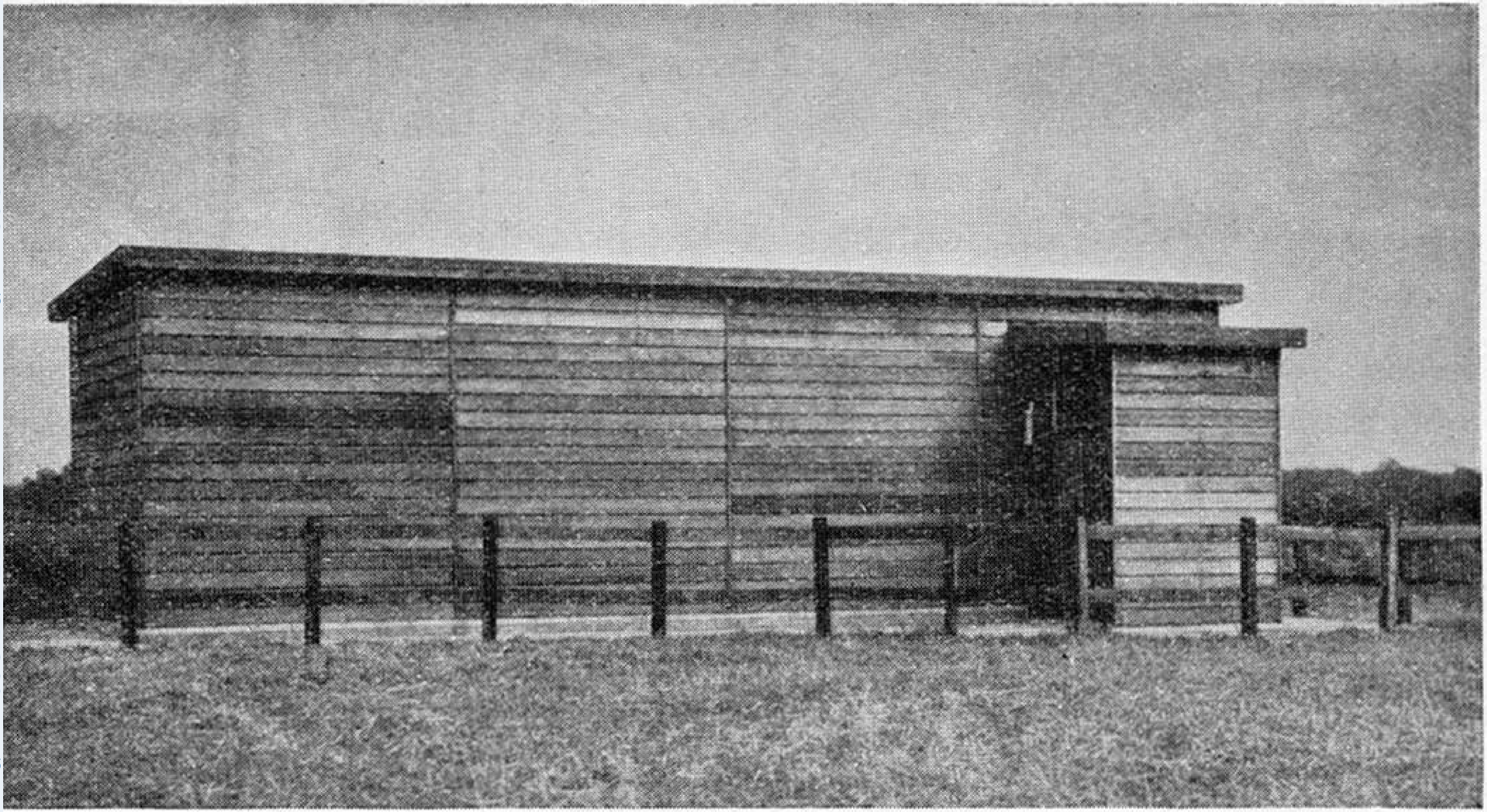

Figure 6. Photograph of magnetometer hut.

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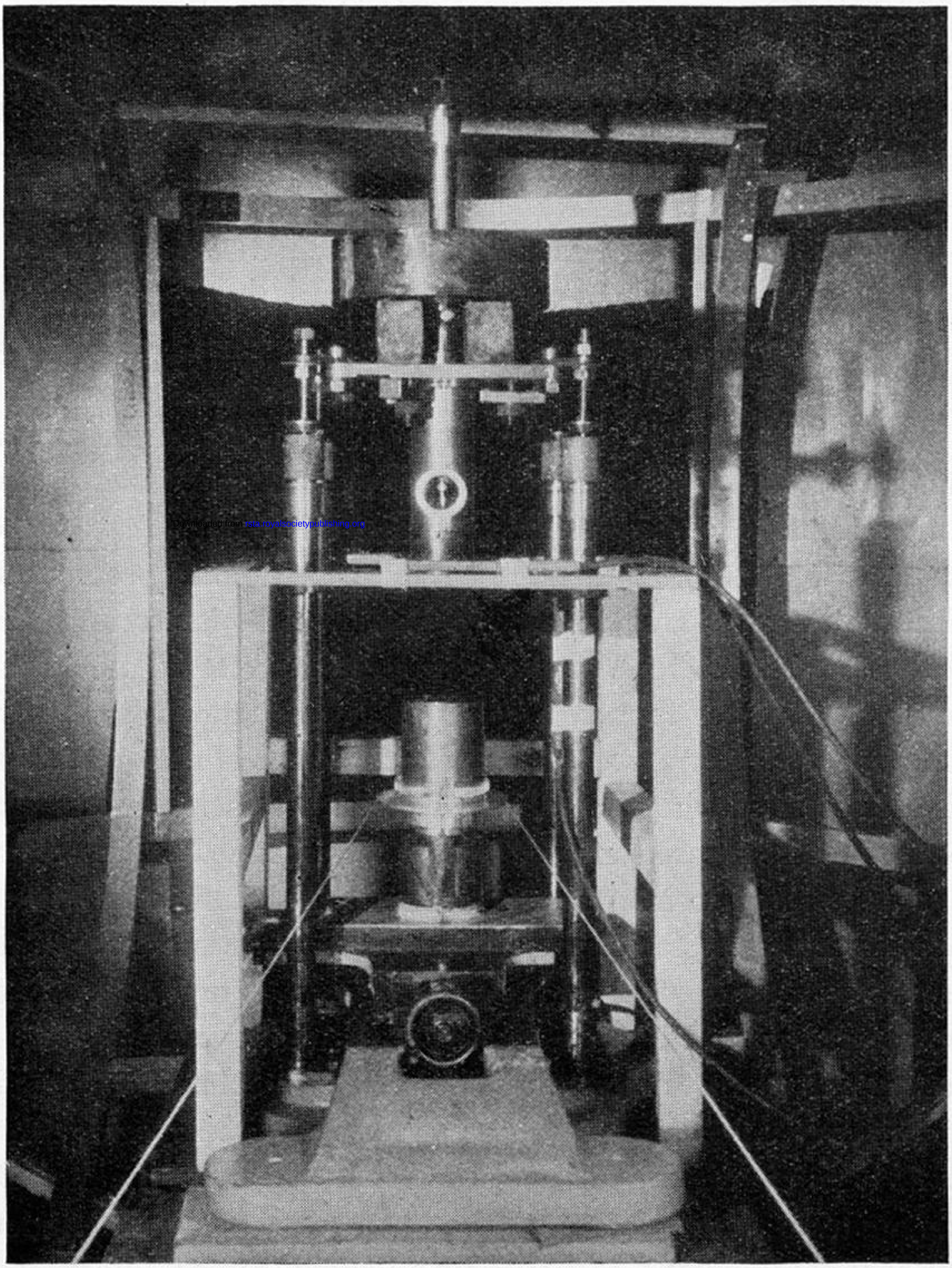

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FIGURE 8. Photograph showing magnetometer case, Julius suspension traversing mechanism and piston carrying 10 cm gold cylinder.

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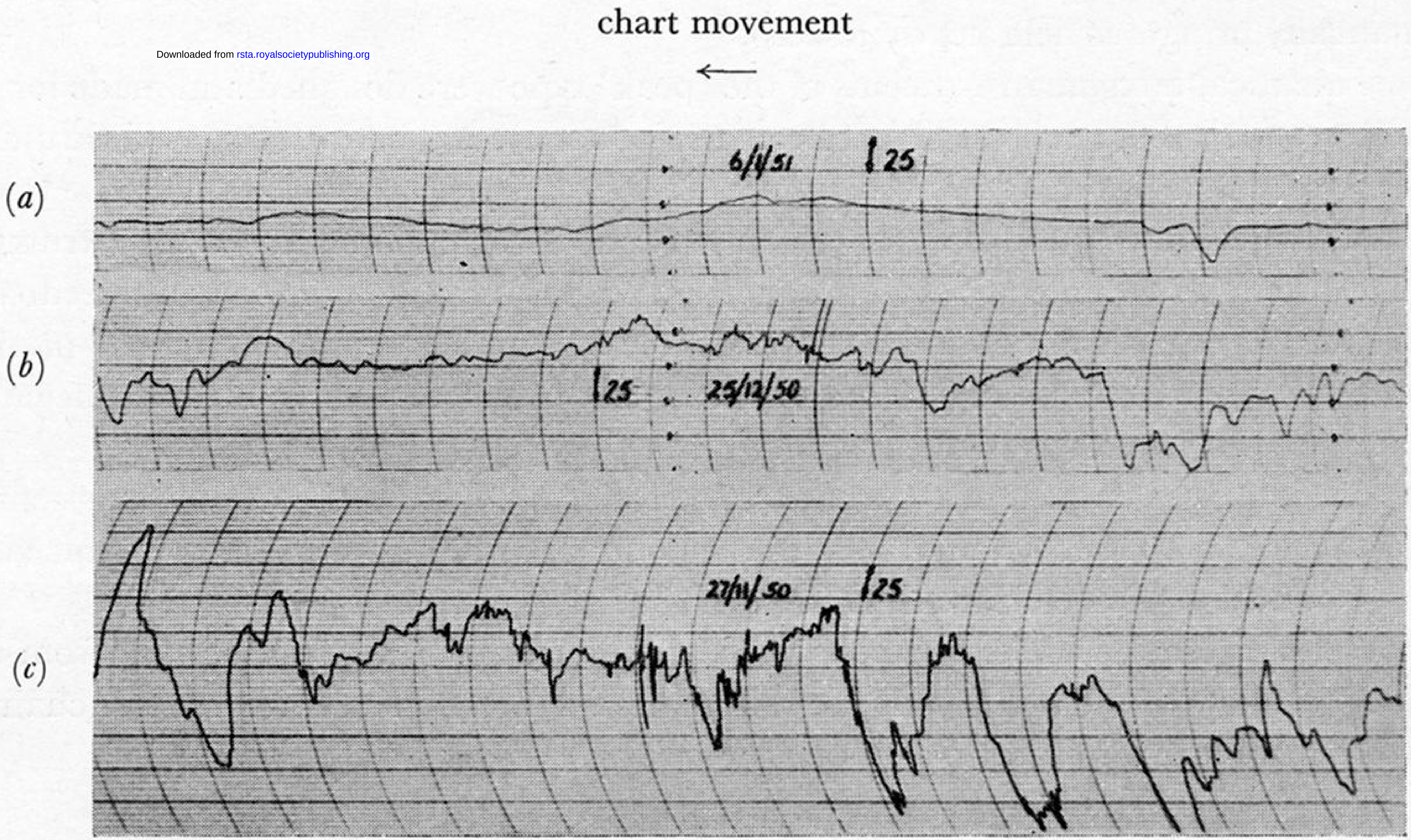

FIGURE 10. Records of east-west fluxgate for three complete days, when the magnetic conditions were (*a*) quiet, (*b*) moderately disturbed, (*c*) very disturbed. The chart speed was 1 division in 1 h. One large division of the vertical scale represents 25 γ.

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